Characterization of Higher Spin Black Holes

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Abstract

Black Holes provide a very useful theoretical laboratory for testing theories of quantum gravity and understanding them in more detail. One of the most basic properties of a black hole is the singularity, a point of infinite curvature located inside a horizon, which is a surface that appears singular in some coordinate frames but is non-singular in others, resulting in an apparent singularity. String Theory is a very ambitious generalization of Einstein's theory of General Relativity, which (apart from the usual spin-2 graviton), also contains an infinite tower of fields with higher spins. In this dissertation the notion of how black holes and their associated singularities have to be adapted in String Theory are examined.

Recently, it has been found that instead of studying the full String Theory, one can learn a lot by looking at so-called Higher-Spin Theories, which contains fields of spin higher than two interacting gravitons in a consistent way. The symmetry group of Higher-Spin theory is larger than the diffeomorphism group of General Relativity, and by allowing these symmetry transformations, the black hole singularities can be removed. What then defines a black hole in Higher Spin Theory? In this regard, the concept of holonomies of an associated gauge field around non-contractible cycles in space-time is investigated, which stays invariant under Higher-spin symmetry transformations.

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Chapter 1

Introduction

1.1 General Relativity

After the invention of Special Relativity by Einstein, for a number of years he tried to create a Lorentz-invariant theory of gravity, but it was not successful. His eventual breakthrough was to replace the Minkowski spacetime (ie. the space where the invariant interval remains invariant after a boost) with a curved spacetime, where the curvature was created and reacted back on energy and momentum. To this end, manifolds are important to continue. A manifold, or differentiable manifold, captures the idea that a space might be curved and have a difficult topology, but in local regions have the same analytical properties as the Euclidean space R^n.

Most of the following can be found from [12], which handles an introductory course into General Relativity. Now, a chart consists of a subset U of a set M which also has a one-to-one map $\phi : U \to R^n$ such that the image $\phi(U)$ is open in R. Then a C^∞ atlas is an indexed collection of charts $\{(U_\alpha, \phi_\alpha)\}$ which satisfies that the union of U_α is equal to the set M, and that the charts are smoothly sewn together. Then it is possible to finally mathematically define a manifold as such: A C^∞ n-dimensional manifold is simply a set M along with a "maximal atlas", one that contains every possible compatible chart. Since the manifold is now defined, it is possible to define functions on it, take their derivatives, set up tensors, consider certain parameterized paths, etc. It is also possible to define volumes of regions and lengths of paths if we incorporate the metric. It could be possible to think of the curvature as depending on the metric, but this is not wholly complete. As shall be discussed shortly, it also requires the creation of a so-called "connection", which will become necessary due to the fact that the partial derivative is not a good tensor operator.

To this end, define the covariant derivative as such,

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu{}_{\mu\lambda} V^\lambda \tag{1.1}$$

where the Γ is known as the connection coefficients. By the same merit, the covariant derivative of a one-form can be expressed as a partial derivative plus some linear transformation,

$$\nabla_\mu \omega_\nu = \partial_\mu \omega_\nu + \tilde{\Gamma}^\lambda{}_{\mu\nu} \omega_\lambda \tag{1.2}$$

where it can be shown that $\Gamma^\lambda{}_{\mu\nu} = -\tilde{\Gamma}^\lambda{}_{\mu\nu}$.

A connection is metric compatible if the covariant derivative of the metric with respect to that connection is everywhere zero. This gives a few valuable properties, like that the inverse metric also has zero covariant derivative

$$\nabla_\rho g^{\mu\nu} = 0 \tag{1.3}$$

and that for a vector field V^λ, a metric-compatible covariant derivative commutes with raising and lowering of indices

$$g_{\mu\lambda} \nabla_\rho V^\lambda = \nabla_\rho (g_{\mu\lambda} V^\lambda) = \nabla_\rho V_\mu. \tag{1.4}$$

From these two properties, both uniqueness and existence can be demonstrated by deriving a manifestly unique expression for the connection in terms of the metric. As such, expand the metric compatibility equation by permuting under three indices:

$$\nabla_\rho g_{\mu\nu} = \partial_\rho g_{\mu\nu} - \Gamma^\lambda{}_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda{}_{\rho\nu} g_{\mu\lambda} = 0 \tag{1.5}$$

$$\nabla_\mu g_{\nu\rho} = \partial_\mu g_{\nu\rho} - \Gamma^\lambda{}_{\mu\nu} g_{\lambda\rho} - \Gamma^\lambda{}_{\nu\mu} g_{\nu\lambda} = 0 \tag{1.6}$$

$$\nabla_\nu g_{\rho\mu} = \partial_\nu g_{\rho\mu} - \Gamma^\lambda{}_{\nu\rho} g_{\lambda\mu} - \Gamma^\lambda{}_{\nu\mu} g_{\rho\lambda} = 0 \tag{1.7}$$

Subtracting the second and third equations from the first, and then using the symmetry of the connection,

$$\partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu} + 2\Gamma^\lambda{}_{\mu\nu} g_{\lambda\rho} = 0 \tag{1.8}$$

And then, by multiplying by $g^{\sigma\rho}$, we solve for the connection

$$\Gamma^\sigma{}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}). \tag{1.9}$$

This is a very important connection, the one on which conventional relativity is based. It is known as the Christoffel or Riemannian connection. Note that even in a curved space it is still possible to make the Christoffel symbols vanish at any one point, because one can make the first derivative of the metric vanish at a point, so the connection coefficients derived from this metric will also vanish.

An interesting feature to discuss at this point is parallel transport. This concept entails the moving of a vector along a path keeping constant all the while, which can be defined whenever a connection exists. The difference between parallel transport in flat spaces and curved spaces is that in a curved space, the result of parallel transporting a vector from one point to another will depend on the path taken between the points. Hence there is no uniquely way to move a vector from one tangent space to another. The equation of parallel transport of a vector can be defined as

$$\frac{d}{d\lambda} V^\mu + \Gamma^\mu{}_{\sigma\rho} \frac{dx^\sigma}{d\lambda} V^\rho = 0, \tag{1.10}$$

however this is not entirely relevant at this stage. So, how is a conventional black hole defined?

1.2 Black Holes, Vielbeins and the Spin Connection

The Riemann tensor will now be investigated to find some symmetries. Examining the Riemann tensor with all lower indices

$$R_{\rho\sigma\mu\nu} = g_{\rho\lambda} R^\lambda{}_{\sigma\mu\nu} \tag{1.11}$$

which has the property that is is antisymmetric in its first two indices

$$R_{\rho\sigma\mu\nu} = -R_{\sigma\rho\mu\nu} \tag{1.12}$$

and is invariant under interchange of the first pair of indices with the second

$$R_{\rho\sigma\mu\nu} = R_{\mu\nu\rho\sigma} \tag{1.13}$$

as well as the vanishing of the antisymmetric part of the last three indices

$$R_{\rho[\sigma\mu\nu]} = 0 \tag{1.14}$$

which would technically imply that the totally antisymmetric part of the tensor would vanish. From this, the Ricci tensor can be defined as:

$$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu} \tag{1.15}$$

which has the property that it is symmetric due to the properties of the Riemann tensor. Then define the Ricci scalar by linking the tensor with the metric:

$$R = R^\mu{}_\mu = g^{\mu\nu} R_{\mu\nu}. \tag{1.16}$$

This is enough information to effectively define the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{1.17}$$

If this equation is equal to zero, this equates to the Einstein equation without matter [6]. This tensor is symmetric due to the symmetry of the Ricci tensor and the metric, and is hence very important in general relativity.

One final piece of formalism is needed to move on to gravitation. Once again, consider the formalism of curvature and connections, but this time use sets of basis vectors in the tangent space which are *not* derived from any coordinate system. So far, it was assumed that a natural basis for the tangent space T_p at a point p is given by the partial derivatives with respect to the coordinates at that point, $\hat{e}_{(\mu)} = \partial_\mu$. Similarly, a basis for the cotangent space $T_p{}^*$ is given by the gradients of the coordinate functions, $\hat{\theta}^{(\mu)} = dx^\mu$. It is possible however to set up any basis that is needed. Hence, imagine at each point in the manifold one introduces a set of basis vectors $\hat{e}_{(a)}$. These must be orthonormal, appropriate to the signature of the manifold. This can also be written as the following, if the canonical form of the metric is used η_{ab}, then the inner product of these basis vectors are

$$g(\hat{e}_{(a)}, \hat{e}_{(b)}) = \eta_{ab}. \tag{1.18}$$

This set of vectors comprising an orthonormal basis is known as a tetrad or vielbein, where depending on the dimension the vielbein can be a vierbein, dreibein, etc. Using this, it is possible to write any vector as a linear combination of basis vectors, specifically

$$\hat{e}_{(\mu)} = e_\mu{}^a \hat{e}_{(a)} \tag{1.19}$$

where the $e_\mu{}^a$ is the vielbein.

By defining the inverse vielbein, the relation between the Minkowski and the Euclidean metric is defined by

$$g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab}. \tag{1.20}$$

Since this introduces a new set of basis vectors, it is necessary to redefine the transformation properties of the set. The only restriction which is necessary is that the orthonormality property be preserved. This is achieved in the flat metric by orhogonal transformations in a Euclidean signature metric, and in a Lorentzian signature metric by Lorentz transformations. So, define the following transformation

$$\hat{e}_{(a)} \to \hat{e}_{(a')} = \Lambda_{a'}{}^a(x)\hat{e}_{(a)} \tag{1.21}$$

where the $\Lambda_{a'}{}^a(x)$ matrices represent position-dependent transformations that leave the canonical form of the metric unaltered.

Now it is necessary to start differentiating. As has been seen previously, the covariant derivative of a tensor results in a partial derivative plus correction terms, one for each index, which involves the tensor and the connection coefficients. For the non-coordinate basis, replace the ordinary coefficients $\Gamma^\lambda{}_{\mu\nu}$ by what is called the spin connection, written as $\omega_\mu{}^a{}_b$. An example for a tensor $X^a{}_b$,

$$\nabla X^a{}_b = \partial_\mu X^a{}_b + \omega_\mu{}^a{}_c X_b - \omega_\mu{}^c{}_b X^a{}_c. \tag{1.22}$$

Doing a comparison between a covariant derivative of a vector X using a purely coordinate basis and mixed basis, one can find the following expression for the spin connection:

$$\omega_\mu{}^a{}_b = e_\nu{}^a e_b{}^\lambda \Gamma^\nu{}_{\mu\lambda} - e_b{}^\lambda \partial_\mu e_\lambda{}^a. \tag{1.23}$$

This is equivalent to the vanishing of the covariant derivative of the vielbein,

$$\nabla_\mu e_\nu{}^a = 0 \tag{1.24}$$

which can be known as the "tetrad postulate".

Now, this leads to the topic of black holes. For a conventional black hole, the following metric is found, the so-called Schwarzschild metric,

$$ds^2 = -(1 - \frac{2GM}{r})dt^2 + (1 - \frac{2GM}{r})^{-1}dr^2 + r^2 d\Omega^2 \tag{1.25}$$

which holds for any spherically symmetric vacuum solution to Einstein's equations; with M used as the conventional Newtonian mass. Note, as $M \to 0$ the Minkowski space is recovered.

Now, looking at the above equation in terms of singularities, it is seen that at $r = 0$ and $r = 2GM$ the metric becomes apparently infinite, the mark of a singularity. Looking at when the curvature becomes infinite, which is measured by the Riemann Tensor, however, it is difficult to say when a tensor becomes infinite, since the components are coordinate dependent. However, from the curvature one can construct various scalar quantities, which are coordinate independent, so they might have meaningful values when they become infinite. The simplest description of such a scalar is the Ricci scalar, $R = g^{\mu\nu}R_{\mu\nu}$, but is also possible to construct higher-order scalars, where if any of them become infinite, it can be regarded that the curvature is a singularity.

In the current setup, it might prove more useful to switch from 4-dimensional to 3-dimensional gravity. In the 4-dimensional case, the solutions to gravity are very complicated. It might be beneficial to first consider the 3-dimensional gravity case, and see if there are any useful properties that might be used in determining higher dimensional cases. Following this, consider the case where there is no cosmological constant, and in this case it was found in [13] that a vacuum solution of (2+1)-dimensional gravity is necessarily flat, and hence there are no black hole solutions. However, in [11], it was shown that for $\Lambda < 0$ there are indeed solutions. This so-called BTZ black hole in the Schwarzschild coordinates is given by the following metric

$$ds^2 = (N^\perp)^2 dt^2 - f^{-2}dr^2 - r^2(d\phi + N^\phi dt)^2 \tag{1.26}$$

with the following for the lapse and shift functions and radial metric

$$N^\perp = f = (-8GM + \frac{r^2}{l} + \frac{16G^2J^2}{r^2})^{\frac{1}{2}}, \qquad N^\phi = -\frac{4GJ}{r^2} \tag{1.27}$$

with ($|J| \leq Ml$). This metric is stationary and axially symmetric, with Killing vectors ∂_t and ∂_ϕ with no other symmetries. Although the space-time is described with a constant negative curvature, it is still a black hole. It has a genuine event horizon at r_+ and if $J \neq 0$, has an inner Cauchy horizon[1] at r_-, with

$${r_\pm}^2 = 4GMl^2(1 \pm [1 - \frac{J}{Ml}^2]^{\frac{1}{2}}). \tag{1.28}$$

The Penrose diagram[2] is essentially identical to that of an asymptotically anti-de Sitter black hole in 3+1 dimensions [8]. The rotating BTZ metric in Euclidean coordinates [7] is given by

$$ds^2 = \frac{(r^2 - {r_+}^2)(r^2 - {r_-}^2)}{r^2l^2}dt^2 + \frac{r^2l^2}{(r^2 - {r_+}^2)(r^2 - {r_-}^2)}dr^2 + r^2(d\phi + i\frac{r_+r_-}{lr^2}dt)^2. \tag{1.29}$$

1.3 Anti-de Sitter Space

At this point it is necessary to define what Anti-de Sitter space, or AdS_n is. It can be defined from [1] that it is a quadric in an $n+1$ deimentional flat spacetime with signature $(n-1, 2)$, i.e. the set of points $(X^1, X^2, ..., X^{n+1})$ which satisfies

$$(X^1)^2 + (X^2)^2 + ... - (X^{n-1})^2 - (X^n)^2 - (X^{n+1})^2 = -1 \tag{1.30}$$

Following this, it is necessary to parametrise AdS_n using the following coordinates:

$$X^1 = \frac{x^1}{x^0}, ..., X^{n-2} = \frac{x^{n-2}}{x^0}, X^n = \frac{x^{n-1}}{x^0}, \tag{1.31}$$

$$X^{n-1} + X^{n+1} = \frac{-1}{x^0}, X^{n-1} - X^{n+1} = \frac{(x^1)^2 + (x^2)^2 + ... + (x^{n-2})^2 - (x^{n-1})^2 + (x^0)^2}{x^0} \tag{1.32}$$

These do not hoewever cover the entire AdS_n, since one must have that $X^{n-1} + X^{n+1} \neq 0$. The metric on the $n+1$ dimensional flat spacetime then induces a Lorentzian metric defined as

$$ds^2 = \frac{1}{(x^0)^2}[(dx^0)^2 + (dx^1)^2 + ... + (dx^{n-2})^2 - (dx^{n-1})^2]. \tag{1.33}$$

These coordinates correlate to the Poincaré coordinates, since the metric becomes singular at $x^0 = 0$, they only ever cover the negative or positive values of x^0. Hence AdS space is conformally flat. It is also possible to make the following substitutions

$$X^n = R\cos t, \qquad X^{n+1} = R\sin t \tag{1.34}$$

[1] A simple introduction to the nature of Cauchy Horizons can be found within [12] and [15].

[2] Penrose Daigrams are defined in [12] and is used for two-dimensional representations of causal relations between different points in space-time.

then (1.25) becomes

$$(X^1)^2 + (X^2)^2 + ... + (X^{n-1})^2 - R^2 = -1. \tag{1.35}$$

This has a metric on the $n+1$ dimensional spacetime as

$$ds^2 = (dX^1)^2 + (dX^2)^2 + ... + (dX^{n-1})^2 - dR^2 - R^2 dt^2. \tag{1.36}$$

From this, if one defines a new coordinate ρ by

$$R = \sqrt{1+\rho^2} \tag{1.37}$$

and noting that $R \geq 1$, then (1.30) becomes

$$(X^1)^2 + (X^2)^2 + ... + (X^{n-1})^2 = R^2 - 1 = \rho^2 \tag{1.38}$$

So, one can hence parametrise $X^1, ..., X^{n-1}$ by $n-2$ angles:

$$X^1 = \rho\cos\alpha_1, X^2 = \rho\sin\alpha_1\cos\alpha_2, ..., X^{n-1} = \rho\sin\alpha_1\sin\alpha_2 ... \sin\alpha_{n-1}. \tag{1.39}$$

This induces a metric in these coordinates as

$$ds^2 = d\rho^2 + \rho^2 d\Omega^2{}_{n-2} - \frac{\rho^2}{1+\rho^2}d\rho^2 - (1+\rho^2)dt^2 = -(1+\rho^2)dt^2 + \frac{d\rho^2}{1+\rho^2} + \rho^2 d\Omega^2{}_{n-2}. \tag{1.40}$$

These then correspond to spherical coordinates on AdS_n, which form a global chart for $\rho \geq 0$; with the time coordinate being periodically identified with a period 2π. By defining these concepts within AdS, the relevence of AdS spaces can now be discussed.

The AdS isometry group is the same as the conformal group of one dimension lower, which can be seen as the starting point for the AdS/CFT correspondence. CFT is short for Conformal Field Theory, and the concept of the correspondence relates the duality of Quantum Field Theory (QFT) with gravity, which is also called the holographic duality or gauge/gravity correspondence [16]. The original context for this duality was discovered for String Theory, where realizing (gauge) field theories on hypersurfaces embedded in a higher dimensional space is quite natural, given the theory contains gravity. Hence, one can extend these concepts to the physics of black holes and quantum gravity. The BTZ black hole is supposedly related to CFT_2, hence the relevance of this topic.

This should be enough information to come to grips with the concepts of the Chern-Simons action.

Chapter 2

Chern-Simons action

2.1 Chern-Simons action from Einstein-Hilbert action

Consider the 1-Form fields from the spin connection and the vielbein:

$$A^a = \omega^a + \frac{i}{l}e^a \qquad \bar{A}^a = \omega^a - \frac{i}{l}e^a \tag{2.1}$$

The e and ω actually have a curved index μ, hence they can be contracted with dx^μ, which would turn them into the 1-forms seen. This then causes the μ index to fall away. One can then think of both the e and ω as $d \times d$ matrices in their flat indices. This is true, since e has only one index, hence it can be thought of as being proportional to the unit matrix (or δ function), while ω is a real matrix. Strictly speaking,the vielbein should be written as ${e_i}^a$ and the spin connection as ${\omega_i}^a{}_b$. Note that any tangent-space indices will be denoted as i, j, k and curved indices as a, b, c. Previously they were called μ, ν, ρ, so note that the notation has been switched.

Geometrically, there exists a smooth manifold M, which in turn has its own tangent bundle T. Also introduce an abstract vector bundle V with dimension d such that its structure group is $SO(d-1,1)$, which therefore implies the existence of a metric η_{ab} which has signature $(-++...+)$. Let the vielbein be the isomorphism between T and V, and the spin connection as an $SO(d-1,1)$ valued connection on V. Now, define the curvature tensor as

$${R_{ij}}^a{}_b = \partial_i {\omega_j}^a{}_b - \partial_j {\omega_i}^a{}_b + [\omega_i, \omega_j]^a{}_b \tag{2.2}$$

or this can simply be written as $R = d\omega + \omega \wedge \omega$.

Now consider the case for when $d = 4$, as is the case at least macroscopically. Then write the Einstein-Hilbert action as

$$I = \frac{1}{2}\int_M \epsilon^{ijkl}\epsilon_{abcd}({e_i}^a {e_j}^b {R_{kl}}^{cd}) \tag{2.3}$$

The above formula can be seen as thus: The expression $e \wedge e \wedge R$ is a four form on M with its values attributed to $V \otimes V \otimes \wedge^2 V$, which maps to $\wedge^4$. Note that V has structure group $SO(3,1)$ which has a natural volume form, hence a section of $\wedge^4 V$ may canonically be considered as a function. Thus the integral $\int e \wedge e \wedge R$ is invariantly defined.

It might be necessary to check whether the above action is indeed the appropriate action for the Einstein theory of gravity. To this end, the metric η_{ab} on V, with the isomorphism ${e_i}^a$ between T and V forms a metric $g_{ij} = {e_i}^a {e_j}^b \eta_{ab}$ on T, which can be seen as an ordinary metric on the manifold M. The connection ω is compatible to this metric since it also has structure group $SO(d-1,1)$. If the Einstein-Hilbert action is varied with respect to ω, then it can be seen that

$$D_i {e_j}^a - D_j {e_i}^a = 0 \tag{2.4}$$

where the covariant derivative is D_i with respect to the connection. This equation implies that ω is torsion free, which is a unique condition to identify this metric compatible connection as the Riemannian connection associated with the metric g_{ij} on M. Finally, varying the action with respect to the vielbein e one can see that

$$e^k{}_a = {R_{ik}}^a{}_b = 0. \tag{2.5}$$

which is equivalent to the vanishing of the Ricci tensor $R_{ij} = {e_j}^b e^k{}_a {R_{ik}}^a{}_b$, which corresponds to the Einstein equations in a vacuum.

Now, why does one try to use the vierbein and the spin connection? Recently, many physicists have tried to combine these two equations (2.1) into a gauge field of the group $ISO(d-1,1)$. By doing this, the aim is to have the spin connection represent the gauge field for Lorentz transformations and the vierbein as the gauge field for translations. However, this causes some problems. In four dimensions, the Einstein-Hilbert action is of the form $\int e \wedge e \wedge (d\omega + \omega^2)$. If e and ω are interpreted as gauge fields, then this is comparable to a gauge action $\int A \wedge A \wedge (dA + A^2)$, but this action does not exist in gauge theory. So in four dimensions this theory makes no sense.

However in three dimensions, other options are possible. For a manifold M of dimension three, the following Einstein-Hilbert action is found:

$$I = \frac{1}{2} \int_M \epsilon^{ijk} \epsilon_{abc} (e_i{}^a (\partial_j \omega_k{}^{bc} - \partial_k \omega_j{}^{bc} + [\omega_j, \omega_k]^{bc})). \tag{2.6}$$

Now, considering the e's and ω's as gauge fields, the general form of this is $AdA + A^3$, which could possibly be linked to a Chern-Simons three form. As such, it is claimed that without a cosmological constant, three-dimensional general relativity is equivalent to a gauge theory with the gauge group ISO(2,1) and a pure Chern-Simons action.

For a gauge group G, the following Chern-Simons interaction is found:

$$I_{CS} = \frac{1}{2} \int_M Tr(A \wedge dA + \frac{2}{3} A \wedge A \wedge A). \tag{2.7}$$

Here, the gauge field A is a Lie-algebra-valued one form.

If one chooses a basis of the lie algebra as $A = A^a T_a$ then the quadratic part of the Chern-Simons interaction becomes

$$Tr(T_a T_b) \cdot \int_M (A^a \wedge dA^b). \tag{2.8}$$

Here $d_{ab} = Tr(T_a T_b)$ plays the role of a metric on the lie algebra and should be non-degenerate.

Now does there exist an invariant and non-degenerate metric on the Lie algebra of $ISO(2,1)$? For the case of $ISO(d-1,1)$ the Lorentz generators are J^{ab} and translations P^a, $a, b = 1, .., d$. A Lorentz-invariant bilinear expression of the form $W = xJ_{ab}J^{ab} + yP_a P^a$ exists for some constants x and y. For $d = 3$, set $W = \epsilon_{abc} P^a J^{bc}$. This is seen to be $ISO(2,1)$ invariant as well as non-degenerate. Hence a reasonable Chern-Simons action for $ISO(2,1)$ will exist.

For $d = 3$, replace J^{ab} with $J^a = \frac{1}{2} \epsilon^{abc} J_{bc}$. The quadratic form of interest in the action is then

$$\langle J_a, P_b \rangle = \delta_{ab}, \qquad \langle J_a, J_b \rangle = \langle P_a, P_b \rangle = 0 \tag{2.9}$$

$ISO(2,1)$ then has the following commutation relations

$$[J_a, J_b] = \epsilon_{abc} J^c, \qquad [J_a, P_b] = \epsilon_{abc} P^c \tag{2.10}$$

$$[P_a, P_b] = 0. \tag{2.11}$$

These relations can be used to construct a gauge theory for $ISO(2,1)$. Let the gauge field be a one form

$$A_i = e_i{}^a P_a + \omega_i{}^a J_a. \tag{2.12}$$

Varying this field under a gauge transformation would then result in

$$\delta A_i = -D_i u, \tag{2.13}$$

where

$$D_i = \partial_i u + [A_i, u]. \tag{2.14}$$

From this, one can calculate the curvature tensor

$$F_{ij} = [D_i, D_j] = P_a(\partial_i e_j{}^a - \partial_j e_i{}^a + \epsilon^{abc}(\omega_{ib} e_{jc} + e_{ib}\omega_{jc})) + J_a(\partial_i \omega_j{}^a - \partial_j \omega_i{}^a + \epsilon^{abc} \omega_{ib}\omega_{jc}). \tag{2.15}$$

which characterizes the connection.

Is it possible to use this Chern-Simons action to define a black hole? If 3-dimensional AdS gravity can be defined using this form, it seems possible to define a spin-2 case.

Chapter 3

Spin-2 case

3.1 CS formulation of three dimensional AdS gravity

The Chern-Simons action we end up using, as proposed by [5] and [9], is the following

$$S = S_{CS}[A] - S_{CS}[\bar{A}] \tag{3.1}$$

where

$$S_{CS}[A] = \frac{k}{4\pi}\int Tr(A \wedge dA + \frac{2}{3}A \wedge A \wedge A). \tag{3.2}$$

The 1-forms A and $\bar{A}$ take the values in the Lie algebra $SL(2,R)$. The Chern-Simons level $k = \frac{l}{4G}$, with l the radius and the Newton constant G. Set $l = 1$ for convenience. The equations of motion corresponding to the vanishing of the fields of the Chern-Simons action are

$$F = dA + A \wedge A = 0, \qquad \bar{F} = d\bar{A} + \bar{A} \wedge \bar{A} = 0 \tag{3.3}$$

These equations impose a flatness condition on the action.

As before, relate these equations with the Einstein equations by forming expressions with the spin connection ω and the vielbein e as

$$A = \omega + ie, \qquad \bar{A} = \omega - ie \tag{3.4}$$

where the indices are dropped for convenience.

The metric is obtained from the vielbeins as follows

$$g_{\mu\nu} = 2Tr(e_\mu e_\nu) \tag{3.5}$$

Now define the asymptotically AdS boundary conditions. To achieve this, choose an explicit basis for the $sl(2,R)$ generators. Define them as

$$L_1 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, \qquad L_{-1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad L_0 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}, \tag{3.6}$$

which obey the commutation relation

$$[L_i, L_j] = (i-j)L_{i+j}. \tag{3.7}$$

It is then possible to reproduce the BTZ metric by choosing appropriate connections, such as

$$A = (e^\rho L_1 - \frac{2\pi\mathcal{L}}{k}e^{-\rho}L_{-1})dz + L_0 d\rho \tag{3.8}$$

$$\bar{A} = (e^\rho L_{-1} - \frac{2\pi\bar{\mathcal{L}}}{k}e^{-\rho}L_1)dz - L_0 d\rho \tag{3.9}$$

where it can be seen that our complex coordinate is $z = \phi + it$ as in the case of the BTZ metric. Note that the BTZ charge dependence $(\mathcal{L}, \bar{\mathcal{L}})$ only arises when ρ is large through the subleading terms. As such, define an asymptotically AdS_3 connection to be one that differs from the above connection by terms that go to zero if ρ is large.

Now, let

$$b = e^{\rho L_0}, \tag{3.10}$$

and

$$a(z) = (L_1 - \frac{2\pi}{k}\mathcal{L}(z)L_{-1})dz \tag{3.11}$$

$$\bar{a}(\bar{z}) = (L_{-1} - \frac{2\pi}{k}\bar{\mathcal{L}}(\bar{z})L_1)d\bar{z} \tag{3.12}$$

then one can write the connections more simply as

$$A = b^{-1}a(z)b + b^{-1}db, \qquad \bar{A} = b\bar{a}(\bar{z})b^{-1} + bdb^{-1}. \tag{3.13}$$

Having defined a spin-2 case, it is natural to want to define these relations for a higher spin. In this regard, the spin-3 case is an obvious choice.

Chapter 4

Spin-3 case

As was seen in the previous chapter, a 3-dimensional black hole (in this case the BTZ black hole) can be written in terms of a spin-2 theory in the Chern-Simons form. This can be extended to the spin-3 case. Here, as in the spin-2 case, we set up generators, however in this case they are $sl(3,R)$. The reasoning of why it is possible to be able to use $sl(3,R)$ by adding a spin-3 field can be seen in [14]. They are given by:

$$L_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad L_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad L_{-1} = \begin{bmatrix} 0 & -2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4.1}$$

$$W_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad W_0 = \frac{2}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{4.2}$$

$$W_{-1} = \begin{bmatrix} 0 & -2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \qquad W_{-2} = \begin{bmatrix} 0 & 0 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{4.3}$$

They satisfy these commutation relations

$$[L_i, L_j] = (i-j)L_{i+j}, \tag{4.4}$$

$$[L_i, W_m] = (2i-m)W_{i+m}, \tag{4.5}$$

$$[W_m, W_n] = -\frac{1}{3}(m-n)(2m^2+2n^2-mn-8)L_{m+n}. \tag{4.6}$$

The generators L_i generate an $sl(2,R)$ subalgebra of $sl(3,R)$. Under this $sl(2,R)$ algebra, the generators W_m form a spin two multiplet. This is called the principal embedding of $sl(2,R)$ into $sl(3,R)$.

Instead of A and $\bar{A}$ it is also possible to determine the specific vielbein and spin connection. By expanding e and ω in a basis of 1-forms dx^μ, the spacetime metric $g_{\mu\nu}$ and spin-3 field are identified as

$$g_{\mu\nu} = \frac{1}{2}Tr(e_\mu e_\nu), \qquad \varphi_{\mu\nu\gamma} = \frac{1}{3!}Tr(e_{(\mu}e_\nu e_{\gamma)}) \tag{4.7}$$

To this end, coupled with the flatness condition mention previously, one can find equations describing a consistent coupling of the metric to the spin-3 field.

Acting on the metric and spin-3 field, the $sl(3,R) \oplus sl(3,R)$ gauge symmetries of the Chern-Simons theory turn into diffeomorphisms along with spin-3 gauge transformations. Under diffeomorphisms, the metric and spin-3 field transform according to the usual tensor transformation rules. The spin-3 gauge transformations are less familiar, as they act nontrivially on the metric and spin-3 fields. If the spin-3 gauge invariance is ignored however, then the theory can be viewed as a particular diffeomorphism invariant theory of a metric and a rank-3 symmetric tensor field.

Consider the connection

$$A(z) = (e^\rho L_1 - \frac{2\pi}{k}e^{-\rho}\mathcal{L}(z)L_{-1} - \frac{\pi}{2k}e^{-2\rho}\mathcal{W}(z)W_{-2})dz + L_0 d\rho \tag{4.8}$$

$$\bar{A}(\bar{z}) = (e^{\rho}L_{-1} - \frac{2\pi}{k}e^{-\rho}\bar{\mathcal{L}}(\bar{z})L_1 - \frac{\pi}{2k}e^{-2\rho}\bar{\mathcal{W}}(\bar{z})W_2)d\bar{z} - L_0 d\rho. \tag{4.9}$$

By setting $\mathcal{W} = \bar{\mathcal{W}} = 0$, the spin-2 case connection is recovered. This can be seen from (4.8) and (4.9), that if $\mathcal{W} = \bar{\mathcal{W}} = 0$, then all the matrices in the connection are within the $sl(2)$ subgroup. This corresponds to spin-2. Moreover, since $A_{\bar{z}} = 0, A_{\rho} = L_0$ and

$$A - A_{AdS} \equiv \mathcal{O}(1) \text{ as } p \to \infty$$

with the same conditions for $\bar{A}$, this connection is asymptotically AdS.

Performing a gauge transformation might be convenient on the connection to obtain

$$a(z) = (L_1 - \frac{2\pi}{k}\mathcal{L}(z)L_{-1} - \frac{\pi}{2k}\mathcal{W}(z)W_{-2})dz \tag{4.10}$$

$$\bar{a}(\bar{z}) = (L_{-1} - \frac{2\pi}{k}\bar{\mathcal{L}}(\bar{z})L_1 - \frac{\pi}{2k}\bar{\mathcal{W}}(\bar{z})W_2)d\bar{z}. \tag{4.11}$$

Now, consider the higher spin charge case, within $sl(3,R) \oplus sl(3,R)$ Chern-Simons theory. In [2] it was proposed that the following is a solution to represent black hole carrying spin-3 charge:

$$a = (L_1 - \frac{2\pi}{k}\mathcal{L}L_{-1} - \frac{\pi}{2k}\mathcal{W}W_{-2})dz - \mu(W_2 - \frac{4\pi\mathcal{L}}{k}W_0 + \frac{4\pi^2\mathcal{L}^2}{k^2}W_{-2} + \frac{4\pi\mathcal{W}}{k}L_{-1})d\bar{z} \tag{4.12}$$

$$\bar{a} = -(L_{-1} - \frac{2\pi}{k}\bar{\mathcal{L}}L_1 - \frac{\pi}{2k}\bar{\mathcal{W}}W_2)d\bar{z} - \bar{\mu}(W_{-2} - \frac{4\pi\bar{\mathcal{L}}}{k}W_0 + \frac{4\pi^2\bar{\mathcal{L}}^2}{k^2}W_2 + \frac{4\pi\bar{\mathcal{W}}}{k}L_1)dz \tag{4.13}$$

To understand the structure of this solution, focus on the a-connection. To add energy and charge density to the $\mathcal{W}_3$ vacuum, add terms involving L_{-1} and W_{-2} to the a_z terms. For black holes, which represent states of thermodynamic equilibrium, the energy and charge should be accompanied by their respective conjugate thermodynamic potentials, which are temperature and spin-3 chemical potentials. The former is incorporated by using the periodicity of imaginary time, while the latter corresponds to a μW_2 term in $a_{\bar{z}}$.

To solve the flatness condition for these solutions, one must show that $[a_z, a_{\bar{z}}] = 0$, which is satisfied for

$$a_{\bar{z}} = -2\mu[(a_z)^2 - \frac{1}{3}Tr({a_z}^2)]. \tag{4.14}$$

This connection can be generalized to more complicated cases, which include Higher Spin Theories hs[λ], which can be seen in [7]. The case of hs[λ] Theories are not however within the scope of this dissertation.

Having been able to effectively define a higher spin case in the Chern-Simons form, it is thus possible to construct multiple solutions to how a black hole is defined, lacking a clear horizon and even distinguishable singularity.

Chapter 5

Holonomies

5.1 Holonomy and Integrability

If it is possible to do higher spin gauge transformations to change the metric, and even make it smooth, how then can a definition of a black hole be generalized to incorporate these properties? The proposal for this question is to look at the holonomy, which is invariant under higher-spin gauge transformations.

For an uncharged BTZ solution the relation between the energy and the temperature is obtained by demanding the absence of a canonical singularity at the horizon in Euclidean signature. For the case of a spin-3 charge is more subtle, hence certain conditions must be set:

(i) The Euclidean geometry is smooth and the spin-3 field is nonsingular at the horizon.

(ii) In the limit $\mu \to 0$ the solution goes smoothly over to the BTZ black hole. In particular, it is required that $\mathcal{W} \to 0$.

(iii) The charge assignment $\mathcal{L} = \mathcal{L}(\tau, \alpha)$ and $\mathcal{W} = \mathcal{W}(\tau, \alpha)$ should arise from an underlying partition function and should hence satisfy

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \frac{\partial \mathcal{W}}{\partial \tau}. \tag{5.1}$$

Note that the metric of the spin-3 theory is not invariant under higher spin gauge transformations and the connection above after a suitable gauge transformation represents a smooth black hole whose charge assignments satisfies the conditions *ii* and *iii*. Hence consider the following holonomy ω around the time circle which is given by

$$\omega = 2\pi(\tau a_z + \tau a_{\bar{z}}) \tag{5.2}$$

It was proposed in [2] that the eigenvalues of the holonomy ω take the fixed values $(0, 2\pi i, -2\pi i)$. It can then be recast as

$$Tr(\omega^2) = -8\pi^2, \qquad Tr(\omega^3) = 0. \tag{5.3}$$

If this holonomy condition is used for the connection, then this becomes

$$0 = -2048\pi^2\mu^3\mathcal{L}^3 + 576\pi k\mu\mathcal{L}^2 - 864\pi k\mu^2\mathcal{W}\mathcal{L} + 864\pi k\mu^3\mathcal{W}^2 - 27k^2\mathcal{W} \tag{5.4}$$

$$0 = 256\pi^2\mu^2\mathcal{L}^2 + 24\pi k\mathcal{L} - 72\pi k\mu\mathcal{W} + \frac{3k^2}{\tau^2} \tag{5.5}$$

with the same format for formulas consisting of the barred quantities replacing the unbarred versions.

Now, solve for $\mathcal{W}$ in the second equation and derive it with respect to τ, then you have a relatable equation for (5.1). Insert this solution into the first equation, then differentiate with respect to α, and τ. Substituting the latter into the first expression for $\frac{\partial \mathcal{W}}{\partial \tau}$, it is found that the expression equals $\frac{\partial \mathcal{L}}{\partial \alpha}$, which is required for the desired integrability condition.

Now define the dimensionless versions of the charge and chemical potential as

$$\zeta = \sqrt{\frac{k}{32\pi\mathcal{L}^3}}\mathcal{W}, \qquad \gamma = \sqrt{\frac{2\pi\mathcal{L}}{k}}\mu. \tag{5.6}$$

Then rewrite (5.4) and (5.5) as

$$1728\gamma^3\zeta^2 - (432\gamma^2 + 27)\zeta - 128\gamma^3 + 72\gamma = 0 \tag{5.7}$$

$$(1 + \frac{16}{3}\gamma^2 - 12\gamma\zeta)\mathcal{L} - \frac{\pi k}{2\beta^2} = 0. \tag{5.8}$$

Now solve the above for the charge ζ and inverse temperature β, which results in

$$\zeta = \frac{1 + 16\gamma^2 - (1 - \frac{16}{3}\gamma^2)\sqrt{1 + \frac{128}{3}\gamma^2}}{128\gamma^3} \tag{5.9}$$

$$\beta = \frac{\sqrt{\frac{\pi k}{2\mathcal{L}}}}{\sqrt{1 + \frac{16}{3}\gamma^2 - 12\gamma\zeta}} \tag{5.10}$$

where a specific branch of ζ is singled out to ensure that for $\gamma \to 0$, $\zeta \to 0$ as well, and in doing so ensures the condition *(ii)*. The difference between this result and the uncharged BTZ case is that here the results limit the values of ζ and γ to

$$\zeta \leq \zeta_{max} = \sqrt{\frac{4}{27}}, \qquad \gamma \leq \gamma_{max} = \sqrt{\frac{3}{16}} \tag{5.11}$$

whereas in the BTZ limit, $\gamma, \zeta \to 0$. Hence, for a given $\mathcal{L}$, a maximal spin-3 charge $\mathcal{W}$ is obtained given by

$$\mathcal{W}^2{}_{max} = \frac{128}{27k}\mathcal{L}^3. \tag{5.12}$$

Note that the maximum values are not directly attributed from the gauge connection or its resulting geometry, but from the holonomy conditions.

5.2 The Black Hole Gauge

Here the aim is to find a suitable gauge transformation to turn the connection (4.12) and (4.13) into a smooth black hole. In particular, it is necessary to see that the smoothness conditions is equivalent to the holonomy conditions. The ρ-dependent connections for (4.12) and (4.13)are denoted as A and $\bar{A}$. They are referred to as being in the wormhole gauge. It is then necessary to relate these connections to the new connections by the $sl(3, R)$ gauge transformations:

$$\mathcal{A} = g^{-1}(\rho)A(\rho)g(\rho)) + g^{-1}(\rho)dg(\rho) \tag{5.13}$$

$$\bar{\mathcal{A}} = g(\rho)\bar{A}(\rho)g^{-1}(\rho) - dg(\rho)g^{-1}(\rho) \tag{5.14}$$

with $g(\rho) \in sl(3, R)$. The metric and spin-3 field corresponding to $(A, \bar{A})$ will be

$$ds^2 = g_{\rho\rho}(\rho)d\rho^2 + g_{tt}(\rho)dt^2 + g_{\phi\phi}(\rho)d\phi^2 \tag{5.15}$$

$$\varphi_{\alpha\beta\gamma}dx^\alpha dx^\beta dx^\gamma = \varphi_{\phi\rho\rho}(\rho)d\phi d\rho^2 + \varphi_{\phi tt}(\rho)d\phi dt^2 + \varphi_{\phi\phi\phi}(\rho)d\phi^3. \tag{5.16}$$

For this to describe a smooth black hole, it is required that this solution has an event horizon at $\rho = \rho_+$. If it is assumed that $g_{rr}(0) > 0$, then this requires $g_{tt}(0) = g'_{tt}(0) = 0$ and $g_{\phi\phi}(0) > 0$, so that after rotating to imaginary time the metric expanded around $r = 0$ will look like $R^2 = S^1$:

$$ds^2 \approx g_{rr}(0)dr^2 - \frac{1}{2}g''_t{}_t(0)r^2dt^2{}_E + g_{\phi\phi}(0)d\phi^2. \tag{5.17}$$

This metric might have a canonical singularity at $r = 0$, so to avoid this, identify $t_E \cong t_E + \beta$ with

$$\beta = 2\pi\sqrt{\frac{2g_{rr}(0)}{-g''_t{}_t(0)}}, \tag{5.18}$$

which enables the switch to Cartesian coordinates, and can be applied to the spin-3 field as well, demanding $\varphi_{\phi rr}(0) = \varphi\prime_{\phi tt}(0) = 0$ and

$$\beta = 2\pi\sqrt{\frac{2\varphi_{\phi rr}(0)}{-\varphi\prime\prime_{\phi tt}(0)}}. \tag{5.19}$$

A single condition is still required for this solution to be completely smooth at the horizon. In terms of Cartesian coordinates, around $r = 0$ one should demand that all functions are infinitely differentiable with respect to the coordinates x and y. If this is not the case, it is expected to see some diverging curvature invariant involving covariant derivatives. Since there is a rotational symmetry, this would imply that the series expansion of the functions should only involve non-negative even powers of r. This gives rise to certain necessary conditions:

$$g_{rr}(-r) = g_{rr}(r), \qquad g_{tt}(-r) = g_{tt}(r), \qquad g_{\phi\phi}(-r) = g_{\phi\phi}(r) \tag{5.20}$$

$$\varphi_{\phi rr}(-r) = \varphi_{\phi rr}(r), \qquad \varphi_{\phi tt}(-r) = \varphi_{\phi tt}(r), \qquad \varphi_{\phi\phi\phi}(-r) = \varphi_{\phi\phi\phi}(r). \tag{5.21}$$

In [4] it was shown that choosing the following values for the vielbeins, one can enforce these symmetry conditions:

$$e_t(-r) = -h(r)^{-1}e_t(r)h(r) \tag{5.22}$$

$$e_\phi(-r) = h(r)^{-1}e_\phi(r)h(r) \tag{5.23}$$

$$e_r(-r) = h(r)^{-1}e_r(r)h(r) \tag{5.24}$$

with $h(r) \in sl(3, R)$, and similar conditions for the spin-connection.

From here, the forms of $g(r)$ and $h(r)$ are examined. Starting from the solutions to the BTZ, a gauge transformation is carried out perturbatively in the charge. This leads to an ansatz shown in the appendix of [3]

$$g(r) = e^{F(r)(W_1 - W_{-1}) + G(r)L_0} \tag{5.25}$$

$$h(r) = e^{H(r)(W_1 + W_{-1})} \tag{5.26}$$

for some functions F,G and H. By Perturbation theory, this ansatz gives a unique solution, shown in [3]. Using this with the holonomy conditions, as well as introducing the parameter C by

$$\zeta = \frac{C-1}{C^{\frac{3}{2}}} \tag{5.27}$$

then the final transformed metric is presented as

$$g_{rr} = \frac{(C-2)(C-3)}{(C-2-cosh^2(r))^2} \tag{5.28}$$

$$g_{tt} = -(\frac{8\pi\mathcal{L}}{k})(\frac{C-3}{C^2})\frac{(a_t + b_t cosh^2(r))sinh^2(r)}{(C-2-cosh^2(r))^2} \tag{5.29}$$

$$g_{\phi\phi} = (\frac{8\pi\mathcal{L}}{k})(\frac{C-3}{C^2})\frac{(a_t + b_t cosh^2(r))sinh^2(r)}{(C-2-cosh^2(r))^2} + (\frac{8\pi\mathcal{L}}{k}(1 + \frac{16}{3}\gamma^2 + 12\gamma\zeta). \tag{5.30}$$

where $a_{t,\phi}$ and $b_{t,\phi}$ are functions of γ and C, which is given by

$$a_t = (C-1)^2(4\gamma - \sqrt{C})^2 \tag{5.31}$$

$$a_\phi = (C-1)^2(4\gamma + \sqrt{C})^2 \tag{5.32}$$

$$b_t = 16\gamma^2(C-2)(C^2 - 2C + 2) - 8\gamma\sqrt{C}(2C^2 - 6C + 5) + C(3C-4) \tag{5.33}$$

$$b_\phi = 16\gamma^2(C-2)(C^2 - 2C + 2) + 8\gamma\sqrt{C}(2C^2 - 6C + 5) + C(3C-4). \tag{5.34}$$

Now, demanding a smooth horizon with (5.17) and (5.18), as well as the definition (5.27), the result is equivalent to (5.7) and (5.8), which are the holonomy conditions evaluated for the gauge connection. Hence the spin-3 charge black hole satisfies the holonomy conditions using the connections (4.12) and (4.13). So, from [3] and [17], since the holonomy conditions are satisfied, there exists a higher spin gauge transformation which makes the solution manifestly a black hole, such that there is an horizon and the higher spin fields are smooth on the horizon.

Chapter 6

Possible Further Study

During the course of this report, a spin-2 and spin-3 case was used. It should be possible to generalize to higher spin cases, even up to a spin-N case. This should be possible if the generators were found for the spin-N case. It is of particular importance for the large N case, since the explicit duality proposal in [14] requires N to be large. It may be the case that the advances in lower dimensions helps to construct black hole solutions in four dimensional higher spin gravity.

Holonomies were used to fix certain parameters of the black hole such that it is consistent with the first law of thermodynamics, and the smoothness in the linearized limit. However, having a more conceptual grasp of this could prove useful in the explicit calculations. The existence of a smooth event horizon in the nonlinear regime is also a possible concern. By assuming there is a spin-3 gauge transformation that exhibits a manifestly non-singular horizon, it might be possible to demonstrate this existence. Perhaps a better understanding of the role of a spin-3 gauge transformation might give a better insight to the phenomena.

Also, it might be worthwhile to consider the supersymmetric generalizations of the higher spin gravities and the Gaberdiel-Gopakumar AdS_3/CFT_2 duality.

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