A study of inventory systems affected by random causes and optimal control of system behaviours.

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Inventory systems are integral factors of business concerns, Manufacturing Industries, warehouses in logistic systems, etc. The inventory serves as safe guard measures to make items available at the time of need in the above mentioned concerns. Inventory managers or planners are often posed with questions

- When to order for more items or when to replenish the inventory so as to make items available?
- How much items should be ordered?

Since placing orders more frequently will increase the costs related to the processing and placing orders and a large sized reorder of items at any time will increase the holding cost in terms of storage, demurrage, etc, one has to find optimal rules to determine the time of ordering and the size of reorder.

Many mathematical models have been proposed in the second half of 20^{th} century and studied in detail. Most of these works considered inventory systems in deterministic setup. When the factors that affect inventory systems are influenced by random causes, the tools developed for deterministic systems cannot be used. Hence in the last few decades stochastic models have been increasingly considered by various researchers.

Yadavalli et al. has, in the recent past, successfully considered new models by assuming broader assumptions that reflect more realistic practical inventory systems and studied those using advanced stochastic tools such as Matrix geometric methods. Some of the models considered are described below:

In all the models we consider inventory systems whose activities are monitored on a continuous time scale. For instance the times at which demand occurs, arrival time of customers, time at which ordered items are received etc, can be any real positive value. The second aspect is the (s, S) ordering policy which is the well known and much practiced in Industry and Business. According to this policy, items are stocked in a storage that can accommodate ‘S’ items. When the stocked items are consumed to meet the demands and at the time of stock level reaching the level, ‘s’, a reorder for (S-s) items are placed so that, if the order is received immediately or before the occurrence of next demand, the stock will be replenished back to S.

Model 1:
The sequence of time points at which demand occurs is assumed to form a renewal process. The lead time to receive the reorder is zero, that is, the order is delivered instantaneously. But the inclusion of ordered items into the stock is implemented in

two different ways:

- Ordered items are brought into the inventory at the time of receipt.
- After the receipt of ordered items, the stock will be brought into the stock only at the next demand epoch.

It may be noted that the two different ways of accepting the reorder will pose complex difficulties and the resultant models are different. It is assumed that one server will be available to process the demands of the customers and the service time is distributed as exponential.

Model 2:
In this model we assume multiple servers, say c servers, to process the demand requests and each take exponentially distributed times. The reorder is delivered after an exponentially distributed times. The demand process is assumed to be a Markovian arrival process (MAP). This is a versatile stochastic process covers a large class of not only renewal processes but also includes correlated arrivals. We also assume another sequence of arrivals according to a different MAP and the associated customers of this process, instead of joining the system, remove any one of waiting customers. These customers may be touts from competing organizations.

Model 3:
In the above two models an infinite population of customer source is inherently assumed. But in this model, the population of customers is assumed to be finite. Hence the usual Poisson, MAP arrivals cannot be considered here. We assume that the arrival process is according to quasi-random. The lead time to receive orders is distributed as exponential. Multiple servers are assumed and they service customers according to an exponential distribution.

In all the above models, the inventory systems are studied using stochastic methods and the steady state results are derived. The total expected costs including setup cost, holding cost etc, is constructed and numerical studies are conducted to get optimal decision rules. The proposed model given below differs from the above in the aspect of optimally controlling the service rates and allowing (multiple) vacation periods for servers.

Proposed Model 4:
We assume Poisson arrivals and service time has exponential distribution. When the customer level is zero or items in the stock are completely depleted or both, the server goes on vacation and spend exponential time. At the end of vacation period, the server is available to provide service only if customer level and stock level has become positive. Otherwise the server goes for another vacation. In this model the service rates are optimally determined.

The four models can be studies in isolation, or in combinations as part of a Masters dissertation or PhD thesis.