University of Pretoria
Department of Economics Working Paper Series

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Working Paper: 2020-98
October 2020

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Abstract

In this paper, we analyse the forecasting ability of a time-varying metric of daily risk aversion for the entire term structure of interest rates of Treasury securities of the United States (US) as reflected by the three latent factors, level, slope and curvature. Using daily data covering the out-of-sample period 22nd June, 1988 to 3rd September, 2020 (given the in-sample period 30th May, 1986 to 21st June, 1988) within a quantiles-based framework, the results show statistically significant forecasting gains emanating from risk aversion for the tails of the conditional distributions of the level, slope and curvature factors at horizons of one-day, one-week, and one-month-ahead. Interestingly, a conditional mean-based model fails to detect any evidence of out-of-sample predictability. Our findings have important implications for academics, bond investors, and policymakers in their quest to better understand the evolution of future movement in US Treasury securities.

Keywords: Yield Curve Factors; Risk Aversion; Out-of-Sample Forecasts

JEL Codes: C22, C53, E43, G12, G17

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1. Introduction

The role of the United States (US) Treasury securities as a traditional "safe haven" cannot be overemphasized enough given their strong ability to provide investors with valuable portfolio diversifications and hedging benefits at times of heightened market stress during which investors' risk appetite turns sour, i.e., risk aversion increases (Kopyl and Lee, 2016; Habib and Stracca, 2017; Hager, 2017), as witnessed during the current pandemic of COVID-19. Given this, a pertinent factor to analyse would be the predictive content of risk aversion for the US government bond market. Understandably, accurate predictability of movement in Treasury securities is an important issue not only for bond investors, but also for policymakers, as an understanding of the evolution of future interest rates helps in the fine tuning of monetary policies.

In spite of the importance of this issue, the forecasting ability of risk aversion for movements of US Treasury securities is limited to the recent work of Çepni et al. (2020a), possibly due to the lack of a robust time-varying measure of risk aversion, given that it is a latent variable and needs to be estimated.[1] These authors find that a metric of time-varying risk aversion (as developed by Bekaert et al. (2019)) obtained from observable financial information which distinguishes time variation in economic uncertainty (the amount of risk) from time variation in risk aversion (the price of risk), predicts (and increase in) monthly US bond premia associated with maturities of 2 to 5 years relative to 1 year, based on a conditional mean-based predictive regression model.[2] Our objective is to build on this study by examining the forecasting ability of risk aversion for the entire term structure of interest rates for the US. For this purpose, we relate risk aversion to the term structure of interest rates, using the well-established framework of Nelson and Siegel (1987). This model summarizes the entire term structure into three latent yield factors, level, slope, and curvature, which, in turn, are considered the only relevant factors that characterise the yield curve (Litterman and Scheinkman, 1991). The factor model of the term structure involving interest rates associated with US Treasury securities of maturities 1 to 30 years in combination with risk aversion,

[1] Related to this line of research, the reader is referred to Laborda and Olmo (2014) and Çepni et al. (2020b) for works analysing the predictability of the US bond premia based on measures of investor sentiment, which is expected to be negatively correlated with risk aversion.

[2] As far as the out-of-sample forecasting experiment is concerned, these authors show that including risk aversion improves the predictive accuracy at all horizons (one- to twelve-months-ahead) for shorter (2- and 3-year) maturity bonds, and at shorter forecast horizons (one- to three-months-ahead) for longer (4- and 5-year) maturity bonds.

enables us to characterize the responses of the yield curve to risk aversion, and calculate the entire yield curve movement in the wake of changes in the risk appetite of investors.

Specifically, we rely on daily estimates of the measure of risk aversion proposed by Bekaert et al. (2019) for the period 30th May, 1986 to 3rd September, 2020, and relate them to the corresponding daily movements of the level, slope and curvature of the yield curve using a quantiles-based framework. The quantile regression model goes beyond the mean-based regression model used by Çepni et al. (2020a), allowing us to test for predictability emanating from risk aversion over the entire conditional distribution of the level, slope and curvature of the yield curve. Given that the period of study involves the zero lower bound (ZLB) situation of interest rates in the US in the wake of the "Great Recession" and following the outbreak of the coronavirus, the use of a quantiles-based framework makes perfect sense, since different quantiles (without having to specify an explicit number of regimes as in a Markov-switching model) can capture the various phases of the 3 latent factors accurately, with the lower, median, and upper quantiles corresponding to low, normal, and high interest rates, respectively. Understandably, high-frequency prediction of the conditional distribution of the term structure of interest rates, unlike the monthly conditional mean-based predictions produced by Çepni et al. (2020a), would allow for the timely and state (regime)-specific design of optimal portfolios involving US government bonds by investors. Furthermore, using the daily information of predictability, policymakers can gauge where the low-frequency real and nominal variables in the economy are headed by feeding the information into mixed-frequency models (Caldeira et al., 2020), given that the entire yield curve is considered a predictor of economic activity (Hillebrand et al., 2018), and, in turn, undertake appropriate monetary policy decisions.

To the best of our knowledge, this is the first paper to study the forecasting ability of risk aversion at daily frequency for the entire conditional distribution of the level, slope and curvature factors characterizing the complete term structure of interest rates of the US. Note that Campbell (2008) points out that the ultimate test of any predictive model (in terms of econometric methodologies and the predictors used) is its out-of-sample performance. Given this, while we present the in-sample analysis (in the Appendix), our focus is on out-of-sample forecastability of the three factors based on the information-content of the time-varying metric of risk aversion.

The rest of the paper is presented in three sections. Section 2 describes the data and the predictive quantile regression approach. Section 3 provides the main results. Section 4 offers some concluding remarks.

2. Data and Econometric Methodology

This section describes the data and the basics of the forecasting model used for our empirical analyses.

2.1. Data

Our analysis covers the period 30th May, 1986 to 3rd September, 2020, with the start and end dates being driven by the availability of data (at the time of writing) on risk aversion and the zero coupon yields respectively. Collected from DataStream, daily zero coupon yields of US Treasury securities with maturities from 1 year to 30 years are used to estimate the yield curve factors based on the dynamic Nelson-Siegel (DNS, hereafter) three-factor model of Diebold and Li (2006). The zero coupon bond yields are based on the work of Gürkaynak et al. (2007), which gives researchers an opportunity to work with a long history of yield curve estimates of the Federal Reserve Board at a daily frequency. Notably, Gürkaynak et al. (2007) use a well-established smoothing method that fits the data and allows the computing of yields for any horizon. The level (L), slope (S) and curvature (C) factors, derived from the DNS model as described in the Appendix, are summarized in Table B1 and plotted in Figure B1 in the Appendix.

Among the dependent variables, the average value of the slope factor is negative, indicating that, on average, yields increase along with maturities. The curvature associated with medium-term maturities has a higher average value than the level factor, which corresponds to long-term yields. This result is in line with Kim and Park (2013) who also use daily bond yields of the US, and is indicative of liquidity issues for bonds with very long maturities. The curvature factor is the most volatile among the three factors, followed by the level and slope. Due to the rejection of the null hypothesis of normality under the Jarque-Bera test, level, slope, and curvature are strongly non-normal, and provide preliminary motivation to look into a quantiles-based approach, to analyse the influence of risk aversion on these variables.

In the paper of Bekaert et al. (2019), the risk aversion coefficient is utility-based, reflecting the time-varying relative risk aversion coefficient of the representative agent in a generalized habitlike model with preference shocks. Given the no-arbitrage framework, asset prices, risk premiums, and physical/risk-neutral variances are exact functions of the state variables, including risk aversion, in the dynamic (exponential) affine model. Since financial variables are observable, the market-wide risk aversion would be spanned by a judiciously-chosen instrument set of asset prices and risk variables. Bekaert et al. (2019) use the generalized method of moments (GMM) to estimate their optimal linear combination given asset moment restrictions that are consistent with the dynamic no-arbitrage asset pricing model. The instrument set includes detrended earnings yield, corporate return spread (Baa-Aaa), term spread (10 year minus 3 month), equity return realized variance, corporate bond return realized variance, and equity risk-neutral variance. An advantage of the risk aversion measure is that, because of its dependence on financial instruments, it can be computed at a daily level, the natural logarithmic value of which is plotted and summarized in Figure B1 and Table B1 respectively. The metric (*RA*) is shown to have its highest value on and around 19th October, 1987, popularly known as Black Monday, and shows sharp increases during the Global Financial Crisis, and the ongoing Coronavirus pandemic.

2.2. Quantile Regression Model

Next, we describe the quantiles-based approach used for our forecasting analysis. Quantile regression was introduced in the seminal paper by Koenker and Bassett (1978), and is a generalization of the median regression analysis to other quantiles. Given the general quantile regression model:

$$y_t = x_t'\beta(\tau) + \varepsilon_t \tag{1}$$

where $\tau \in (0,1)$ and ε_{t+1} are assumed independent derived from an error distribution $g_\tau(\varepsilon)$ ($\int_{-\infty}^{0} g_\tau(\varepsilon)d\varepsilon = \tau$) with the τ-th quantile equal to 0. The coefficients of the τ^{th} conditional quantile distribution are estimated as:

$$\widehat{\beta}\,(\tau) = \arg\min \sum_{t=1}^{\tau} \left(\tau - 1_{\{y_t < x_t'\beta(\tau)\}}\right) |\, y_t - x_t'\,\beta(\tau)| \tag{2}$$

where the quantile regression coefficient $\beta(\tau)$ determines the connection between the vector of independent variables and the τ^{th} conditional quantile of the dependent variable, with $1_{\{y_t < x_t'\beta(\tau)\}}$ being the usual indicator function. In our case, to account for predictability, based on the lags chosen by the Schwarz Information Criterion (SIC), we have: x_t'=[1 L_{t-1} S_{t-1} C_{t-1} RA_{t-1}], with y_t being L_t, S_t, or C_t.

3. Empirical Results

Though the focus of our analysis is the out-of-sample forecasting, in Figure B2 we plot the response of L_t, S_t, and C_t to RA_{t-1} over the conditional distribution of the three latent factors, with strong evidence of predictability from risk aversion observed around the lower and upper quantiles.[3,4] Next, we turn our attention to the forecasting exercise, since in-sample predictability does not guarantee that the same will hold over an out-of-sample period (Rapach and Zhou, 2013). To design the experiment, in particular to determine the in- and out-of-sample splits, we use the powerful *UDmax* and *WDmax* tests of Bai and Perron (2003), to detect 1 to *M* structural breaks in equation (1), allowing for heterogenous error distributions across the breaks. When we apply these tests, we detect five, four and four breaks for the equations associated with L_t, S_t, and C_t respectively at: 22/6/1988, 24/5/1991, 16/2/1993, 7/11/1994, 24/7/1996; 22/6/1988, 21/5/1992, 14/7/1994, 11/6/1996; 22/6/1988, 16/2/1993, 7/11/1994, 15/1/2009. Given that the earliest break in all three cases is found at 22nd June, 1988, the out-of-sample period starts from that date, over which the model is recursively estimated to produce forecast for horizons (h) = 1-, 5- and 22-day-ahead, corresponding to one-day, one-week, and one-month-ahead forecasts respectively. In Table 1, we report the relative mean squared forecast errors (RMSFEs) obtained from equation (1) with and without risk aversion, derived under the conditional mean- and quantiles-based estimations (over the quantile range 0.05 to 0.95 with an increment of 0.05). Understandably, a value of less than unity for the RMSFE, would indicate that *RA* adds value to forecasts of *L*, *S* and *C*.

As can be seen from Table 1, the conditional mean-based model indicates no evidence of forecastability due to the metric of risk aversion for the three latent factors. However, for the quantile model, consistent with the in-sample evidence, the forecastability of the level, slope and curvature factors due to risk aversion is concentrated at the tails of the conditional

[3] The ordinary least squares (OLS) estimation of the model in equation (1), with Newey and West (1987) heteroscedasticity and autocorrelation corrected (HAC) standard errors, provides evidence of predictability for level and slope, but not the curvature factor. Complete details of these results are available upon request from the authors.

[4] It must be noted that the safe-haven property of the US Treasury securities, which would imply a decline in the yield (rise in returns) following an increase in risk aversion, tends to hold towards the upper end of the conditional distribution of the three factors.

distributions of the three latent term structure factors. More importantly, when we use the *MSE-F* test of McCracken (2007) to compare the forecast accuracy of the model with risk aversion relative to the one without, i.e., the nested model, the forecasting gains at the tails are found to be statistically significant. Our results tend to suggest that, when the US Treasury securities market is performing in its normal mode, i.e., on and around the median, risk aversion does not contain any predictive information. However, when the market is in bearish or bullish phases, investors use the information contained in the risk aversion variable to improve the profitability position of their portfolio that comprises US Treasury securities.

[INSERT TABLE 1]

4. Conclusion

In this paper, we analyse the forecasting ability of a time-varying metric of daily risk aversion for the entire term structure of interest rates of US Treasury securities, as reflected by the three latent factors, level, slope and curvature. Based on data covering the out-of-sample period 22nd June, 1988 to 3rd September, 2020 (given the in-sample period 30th May, 1986 to 21st June, 1988), we find that a conditional mean-based model fails to detect any evidence of out-of-sample predictability running from risk aversion to the three yield curve factors. However, when we rely on a quantiles-based framework, and reconduct the forecasting exercise, we detect evidence of statistically significant forecasting gains emanating from risk aversion for the tails of the conditional distributions of the level, slope and curvature factors at horizons of one-day, one-week, and one-month-ahead.

Understandably, our findings at high-frequency, i.e., daily data, have multi-dimensional implications. The observation that risk aversion contains predictive information over the evolution of future interest rates in a quantiles-based set-up can help policymakers fine-tune their monetary policy models (Hkiri et al., forthcoming), given that risk aversion forecasts the slope factor of the yield curve, which captures movements of short-term interest rates. Moreover, bond investors can improve their investment strategies by exploiting the role of risk aversion in their interest-rate prediction models, while risk managers can develop asset allocation decisions conditional on the level of the risk aversion, particularly when the bond market is in its extreme phases. Finally, researchers may utilize our findings to explain deviations from asset-pricing models by embedding risk aversion in their pricing kernels, which, however, needs to account for the entire conditional distribution, rather than just the conditional mean.

While we concentrate on US Treasury securities given their global dominance in the sovereign bond market, as part of future research it would be interesting to extend our analysis to the term structure factors associated with the government bond markets of other developed and emerging countries, given that the risk aversion index of the US can be used as a proxy for global risk appetite (Demirer et al., 2018).

References

Bai, J., and Perron, P. (2003). Computation and analysis of multiple structural change models. Journal of Applied Econometrics, 18, 1-22.

Bekaert, G., Engstrom, E., and Xu, N.R. (2019). The time variation in risk appetite and uncertainty. NBER Working Paper No. 25673.

Caldeira, J.F., Gupta, R., Suleman, M.T., and Torrent, H.S. (2020). Forecasting the Term Structure of Interest Rates of the BRICS: Evidence from a Nonparametric Functional Data Analysis. Emerging Markets Finance and Trade. DOI: https://doi.org/10.1080/1540496X.2020.1808458.

Campbell, J.Y. (2008). Viewpoint: estimating the equity premium. Canadian Journal of Economics, 41, 1–21.

Çepni, O., Demirer, R., Gupta, R. and Pierdzioch, C. (2020a). Time-varying risk aversion and the predictability of bond premia. Finance Research Letters, 34, 101241.

Çepni, O., Guney, E., Gupta, R. and Wohar, M.E. (2020b). The role of an aligned investor sentiment index in predicting bond risk premia of the U.S. Journal of Financial Markets. DOI: https://doi.org/10.1016/j.finmar.2020.100541.

Demirer, R., Omay, T., Yüksel, A., and Yüksel, A. (2018). Global Risk Aversion and Emerging Market Correlations. Economics Letters, 173, 118–121.

Diebold, F.X., and Li, C. (2006). Forecasting the term structure of government bond yields. Journal of Econometrics, 130(2), 337-364.

Gürkaynak, R.S., Sack, B., and Wright, J.H. (2007). The U.S. Treasury yield curve: 1961 to the present? Journal of Monetary Economics, 54(8), 2291-2304.

Habib, M.M., and Stracca L. (2015). Is There a Global Safe Haven? International Finance, 18(3), 281-298.

Hager, S. B. (2017). A global bond: Explaining the safe-haven status of US Treasury securities. European Journal of International Relations, 23(3), 557-580.

Hillebrand, E., Huang, H., Lee, T-H., and Li, C. (2018). Using the Entire Yield Curve in Forecasting Output and Inflation. Econometrics, 6, 40.

Hkiri, B, Cunado, J., Balcilar M., and Gupta, R. (Forthcoming). Time-Varying Relationship between Conventional and Unconventional Monetary Policies and Risk Aversion: International Evidence from Time- and Frequency-Domains. Empirical Economics.

Kim, H., and Park, H. (2013). Term structure dynamics with macro-factors using high frequency data. Journal of Empirical Finance, 22, 78-93.

Koenker, R. and Bassett, G. (1978). Regression quantiles. Econometrica, 46(1), 33-50.

Kopyl, K.A., and Lee, J.B-T. (2016). How safe are the safe haven assets? Financial Markets and Portfolio Management, 30(4), 453-482.

Laborda, R. and Olmo, J. (2014). Investor sentiment and bond risk premia. Journal of Financial Markets, 18, 206-233.

Litterman, R.B., and Scheinkman, J. (1991). Common factors affecting bond returns. Journal of Fixed Income, 1(1), 54–61.

McCracken, M.W. (2007). Asymptotics for out of sample tests of Granger causality. Journal of Econometrics, 140, 719-752.

Nelson, C.R., and Siegel, A.F. (1987). Parsimonious modeling of yield curves. Journal of Business, 60(4), 473-489.

Newey, W.K., and West, K.D. (1987). A simple positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix. Econometrica, 55, 703-708.

Rapach, D.E., and Zhou, G. (2013). Forecasting stock returns. Handbook of Economic Forecasting, 2 (Part A), Graham Elliott and Allan Timmermann (Eds.), Amsterdam: Elsevier, 328−383.

Table 1. Relative Mean Squared Forecast Errors (RMSFEs)

		Level			Slope			Curvature		
		h=1	*h* =5	*h* =22	*h*=1	*h* =5	*h* =22	*h*=1	*h* =5	*h* =22
Quantile	OLS	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0000	1.0000	1.0000
	0.05	0.9994***	0.9986***	0.9986***	0.9995**	0.9996**	0.9997**	1.0000	0.9999	1.0001
	0.1	0.9990***	0.9990***	0.9990***	0.9989***	0.9989***	0.9987***	0.9995**	0.9995***	0.9997**
	0.15	0.9984***	0.9983***	0.9978***	0.9996**	0.9995**	0.9995*	0.9998**	0.9997**	0.9998**
	0.2	0.9992***	0.9991***	0.9992***	1.0005	1.0004	1.0007	0.9997**	0.9997**	0.9997**
	0.25	0.9995***	0.9995***	0.9995***	1.0001	1.0002	1.0005	0.9998**	0.9997**	0.9997**
	0.3	0.9998**	0.9998*	0.9997**	1.0003	1.0002	1.0003	0.9999	0.9999	0.9999
	0.35	0.9998*	0.9999*	0.9998**	1.0003	1.0003	1.0003	1.0000	0.9999	1.0000
	0.4	0.9999*	0.9999	0.9999*	1.0004	1.0005	1.0005	1.0000	1.0000	1.0000
	0.45	0.9999	1.0000	1.0000	1.0003	1.0002	1.0002	1.0000	1.0000	1.0000
	0.5	0.9999	0.9999	0.9999*	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000
	0.55	1.0001	1.0001	1.0001	0.9999	0.9999	0.9999*	1.0000	1.0001	1.0000
	0.6	1.0001	1.0002	1.0002	0.9999*	0.9998*	1.0000	1.0001	1.0001	1.0001
	0.65	1.0004	1.0003	1.0003	0.9998**	0.9999*	0.9998*	1.0002	1.0002	1.0002
	0.7	1.0003	1.0002	1.0003	0.9998**	0.9997**	0.9998*	1.0003	1.0001	1.0004
	0.75	1.0001	1.0001	1.0004	0.9989***	0.9988***	0.9988***	1.0002	1.0003	1.0003
	0.8	1.0004	1.0004	1.0003	0.9983***	0.9984***	0.9975***	0.9999*	0.9998*	0.9999*
	0.85	0.9997**	0.9996**	0.9996**	0.9996**	0.9997**	0.9995***	0.9997**	0.9998**	0.9999*
	0.9	0.9995***	0.9994***	0.9994***	0.9994***	0.9993***	0.9993***	0.9990***	0.9992***	0.9992***
	0.95	0.9994***	0.9994***	0.9995*	1.0003	1.0000	1.0003	0.9993***	0.9991***	0.9991***

Note: RMSFEs correspond to the MSFE of the model (equation (1)) with risk aversion relative to one without; ***, **, and * indicate significance of the MSE-F test statistic at 1%, 5%, and 10% levels of significance respectively.

APPENDICES:

APPENDIX A:

Extracting Yield Curve Factors

The DNS model of Diebold and Li (2006) is employed to decompose the yield curve of US zero coupon Treasury securities into three latent factors. The DNS with time-varying parameters is given by:

$$r_t(\tau) = L_t + S_t\left(\frac{1-exp^{-\lambda\tau}}{\lambda\tau}\right) + C_t\left(\frac{1-exp^{-\lambda\tau}}{\lambda\tau} - exp^{-\lambda\tau}\right) \quad \text{(A1)}$$

where r_t represents the yield rate at time t and τ is the time to maturity. The factor loading of L_t is equal to 1 and loads equally for all maturities (1 to 30 years). A change in L_t leads to an equal change in all yields; therefore, L_t is the level factor representing the movements of long-term yields. The loading of S_t starts at 1 and monotonically decays to zero. As for S_t, it changes the slope of the yield curve. Thus, S_t is the slope factor that mimics the movements of short-term yields. The loading for C_t starts at 1 and decays to 0, with a hump in the middle. An increase in C_t leads to an increase in the yield curve curvature; thus C_t is the curvature factor that mimics medium-term yield movements. The DNS model follows a vector autoregressive (VAR) process and is modelled in state-space form using the Kalman filter. The measurement equation relating the yields and latent factors is given by:

$$\begin{pmatrix} r_t(\tau_1) \\ r_t(\tau_2) \\ \vdots \\ r_t(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & \left(\frac{1-exp^{-\tau_1\lambda}}{\tau_1\lambda}\right) & \left(\frac{1-exp^{-\tau_1\lambda}}{\tau_1\lambda} - exp^{-\tau_1\lambda}\right) \\ 1 & \left(\frac{1-exp^{-\tau_2\lambda}}{\tau_2\lambda}\right) & \left(\frac{1-exp^{-\tau_2\lambda}}{\tau_2\lambda} - exp^{-\tau_2\lambda}\right) \\ \vdots & \vdots & \vdots \\ 1 & \left(\frac{1-exp^{-\tau_n\lambda}}{\tau_n\lambda}\right) & \left(\frac{1-exp^{-\tau_n\lambda}}{\tau_n\lambda} - exp^{-\tau_n\lambda}\right) \end{pmatrix}' f_t + \begin{pmatrix} u_t(\tau_1) \\ u_t(\tau_2) \\ \vdots \\ u_t(\tau_1) \end{pmatrix}, \; u_t \sim N(0, R) \quad \text{(A2)}$$

The transition equation relating the dynamics of the latent factors is represented by:

$$\tilde{f}_t = \Gamma \tilde{f}_{t-1} + \eta_t \qquad \eta_t \sim N(0, G) \quad \text{(A3)}$$

where $r_t(\tau)$ and u_t are $m \times 1$ dimensional vectors for yield rates with given maturities (1 year to 30 years) and the residual terms, respectively. The coefficient matrix in equation (A2) follows the structure introduced by Nelson and Siegel (1987), $f_t = [L_t, S_t, C_t]$ is a 3×1 dimensional vector, containing the yield rate shape parameters that vary over time. In equation (A3), $\tilde{f}_t = f_t - \overline{f}$ is the demeaned time-varying shape parameter matrix, Γ represents the dynamic relationship across shape parameters, η_t is a 3×1 dimensional error vector assumed to be independent of u_t, G is a $m \times m$ dimensional diagonal matrix, and R is a 3×3 dimensional variance-covariance matrix that allows the latent factors to be correlated.[5]

[5] Given that the estimation procedure is beyond the scope of this paper, interested readers are referred to Diebold and Li (2006). In parallel, the authors of this paper can share more details of the parameter estimates of the model upon request.

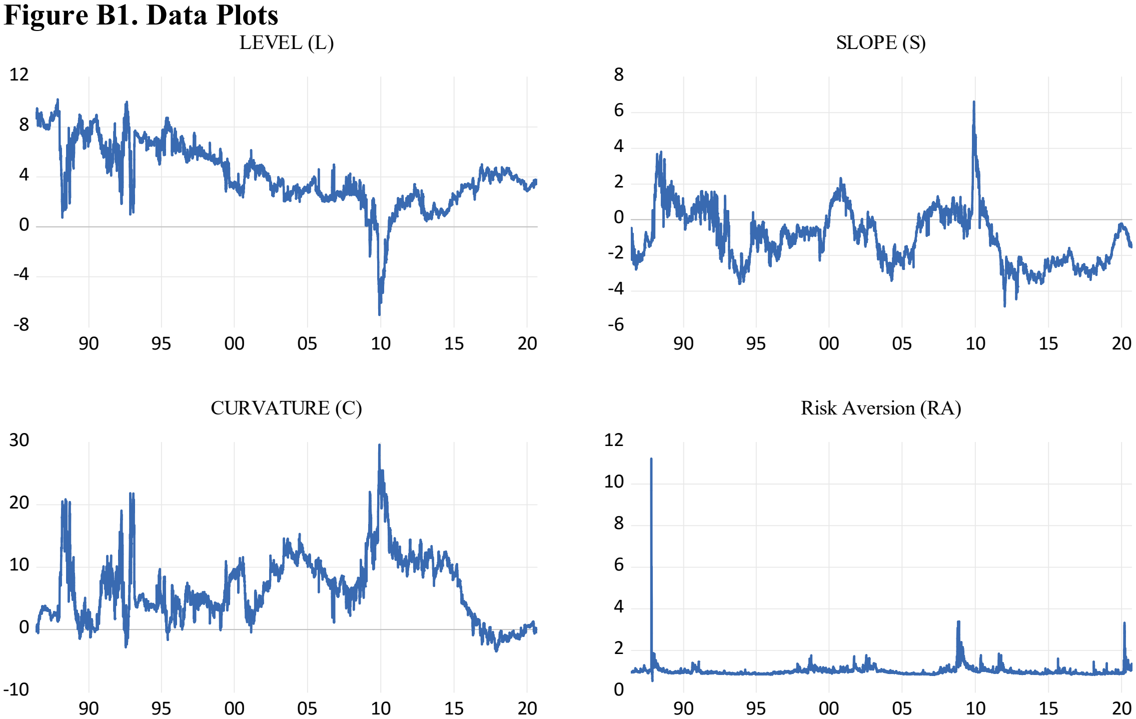
APPENDIX B:

Table B1. Summary Statistics

	Variable			
Statistic	Level (*L*)	Slope (*S*)	Curvature (*C*)	Risk Aversion (*RA*)
Mean	4.1363	-0.9530	6.5158	1.0031
Median	3.7590	-0.9680	6.0244	0.9500
Maximum	10.2120	6.6251	29.6086	11.2000
Minimum	-7.0564	-4.8584	-3.4930	0.5280
Std. Dev.	2.5208	1.5458	5.2729	0.2524
Skewness	-0.2667	0.5256	0.5970	16.2026
Kurtosis	3.8429	3.2658	3.3820	519.5581
Jarque-Bera	353.9813***	418.3165***	559.0019***	95299157.0000***
Observations	8,538			

Note: RA: Natural logarithm of risk aversion; *** indicates rejection of the null hypothesis of normality at the 1% level of significance.

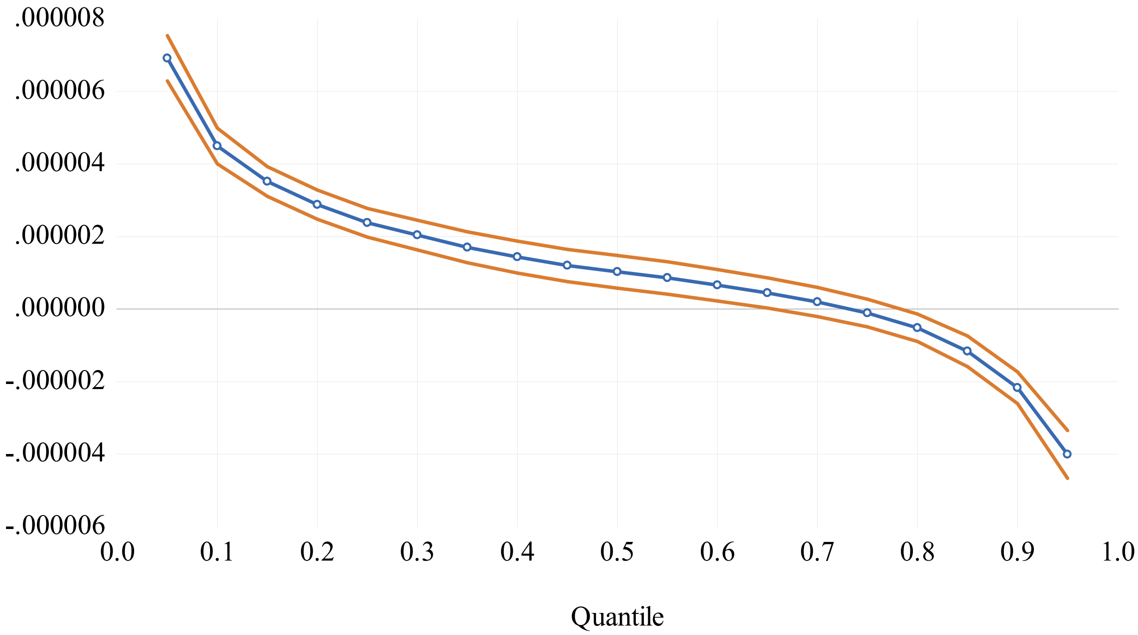
Figure B1. Data Plots



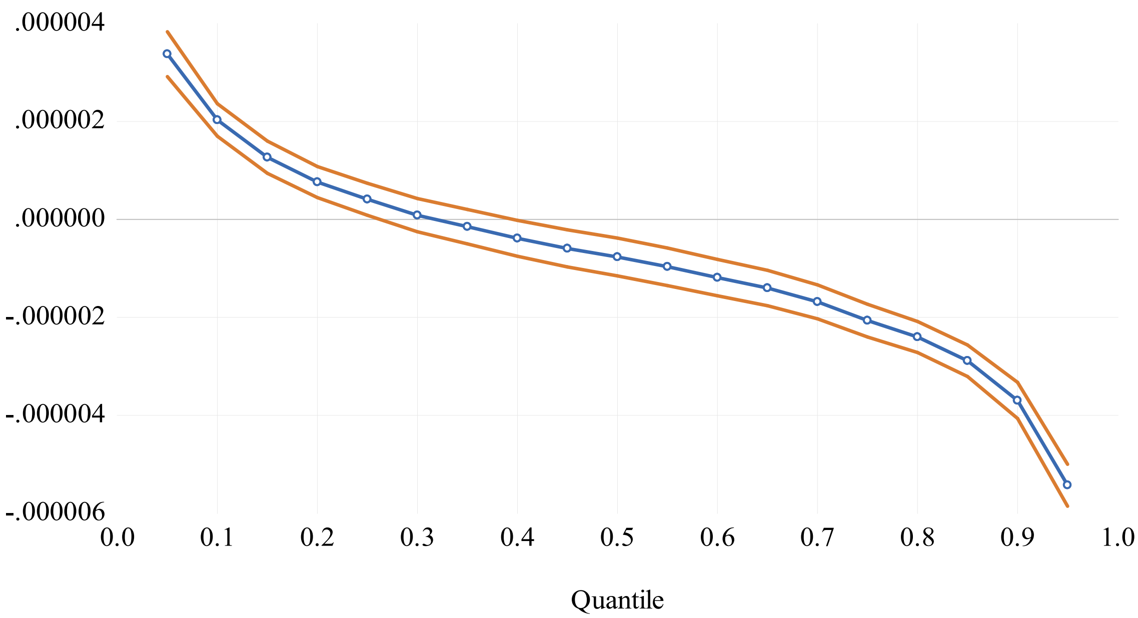
Note: See Notes to Table A1.

Figure B2. Response of the Yield Curve Factors to Lagged Risk Aversion (*RA*):

(a) In-Sample Predictability of Level (L)



(b) In-Sample Predictability of Slope (S)



(c) In-Sample Predictability of Curvature (C)