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Investors' Uncertainty and Forecasting Stock Market Volatility

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Abstract

This paper examines if incorporating investors' uncertainty, as captured by the conditional volatility of sentiment, can help forecasting volatility of stock markets. In this regard, using the Markov-switching multifractal (MSM) model, we find that investors' uncertainty can substantially increase the accuracy of the forecasts of stock market volatility according to the forecast encompassing test. We further provide evidence that the MSM outperforms the Dynamic Conditional Correlation-Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model.

Keyword: Investors' uncertainty, Stock market risk, MSM, Volatility forecasting.

1 Introduction

In the wake of the recent global financial crisis, measuring (investors') uncertainty, a latent variable, and analyzing its impact on stock market movements and the broader economy of the United States (US) (and internationally) has grown in importance (see for example, Chuliá et al. (2017a, b), Gupta et al. (2018, 2020a) for detailed literature reviews in this regard). As indicated above, uncertainty is an unobservable concept, and hence, needs to be measured. Given this, besides the various alternative metrics of uncertainty associated with financial markets (such as the implied-volatility indices (popularly called the VIX), realized volatility, idiosyncratic volatility of equity returns, corporate spreads), there are primarily three broad approaches to quantify uncertainty (Gupta et al., 2019, 2020b): (1) A news-based approach, with the main idea behind this method being to perform searches of major newspapers for terms related to economic and policy uncertainty, and then to use the results to construct indices of uncertainty; (2) Derive measures of uncertainty from stochastic-volatility estimates of various types of small and large-scale structural models related to macroeconomics and finance, and; (3) Uncertainty obtained from dispersion of professional forecaster disagreements.

In terms of alternative metrics of uncertainty, Escobari and Jafarinejad (2019) proposed a measure of investors' uncertainty by estimating the conditional volatility of a widely used measure of investor sentiment namely, the bull-bear spread in the American Association of Individual Investors (AAII) Sentiment Survey. The measure aims to capture the dispersion in expectations of market participants, which Escobari and Jafarinejad (2019) interpret as investors' uncertainty about the future, which in turn is then linked with US stock market risk, i.e., conditional volatility. The authors find that the conditional volatility of major stock market indices (Center for Research in Security Prices, CRSP; New York Stock Exchange, NYSE; American Stock Exchange, AMEX; National Association of Securities Dealers Automated Quotations, NASDAQ; Dow Jones Industrial Average, DJIA; and the S&P500) is positively related with the metric of investors' uncertainty, suggesting that uncertainty possibly induce systematic risk (Lee et al., 1991), and increase the volatility of stock returns (Andrei and Hasler, 2015).

We aim to extend the work of Escobari and Jafarinejad (2019) by analyzing the out-of-sample predictive ability of the sentiment-based measure of investors' uncertainty for the volatility of the above mentioned stock returns. This is an important issue, since in-sample predictability does not guarantee forecasting gains (Ben Nasr et al., 2016), with Campbell (2008) stressing that the ultimate test of any predictive model (in terms of the econometric methodologies and the predictors used) is in its out-of-sample performance. Moreover, appropriate modeling and accurate forecasting of the process of volatility has ample implications for portfolio selection, the pricing of derivative securities and risk management (Poon and Granger, 2003; Rapach et al., 2008).

From an econometric perspective, while we follow Escobari and Jafarinejad (2019) to account for the joint dynamics of sentiment, uncertainty, returns and risk using the Dynamic Conditional Correlation-Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) method (Engle, 2002) as our benchmark model, we also consider univariate and bivariate versions of the Markov-switching multifractal (MSM) model of Calvet et al (2006). Note that, research on long memory and structural changes in volatility has discussed the connection between these phenomena, and have suggested that, in fact, volatility persistence may be due to switching of regimes in the volatility process (Diebold, 1986; Lamoureux and Lastrapes, 1990). Hence, it could be very difficult to distinguish between true and spurious long memory processes. This ambiguity motivates us to consider the MSM framework, which, despite allowing for a large number of regimes, is more parsimonious in parameterization than other regime-switching models. Moreover, it is well-known to give rise to apparent long memory over a bounded interval of lags (Calvet and Fisher, 2004) and it has limiting cases in which it converges to a 'true' long memory process.

To the best of our knowledge, this is the first attempt in forecasting the volatility process for major stock indices based on investors' uncertainty using bivariate volatility models that capture long-memory, structural breaks and the fact that structural breaks can lead to the spurious impression of long-memory. In addition, note that the bivariate approach also allows us to measure investors' uncertainty, derived from conditional volatility of sentiment, simultaneously with the volatility of stock markets. In this regard, our study is also different from the few studies that exist associated with in-sample and out-of-sample predictability of stock market volatility based on primarily the new papers-based measure of economic uncertainty, derived from outside the econometric model associated with volatility (see for example, Liu and Zhang (2015), Su et al. (2017, 2019), Fang et al. (2018), Li et al. (2019)). Modeling joint dynamics is important, given the findings of recent studies (see for example, Mumtaz and Theodoridis (2020), Ludvigson et al. (forthcoming)), which indicates that uncertainty is in fact endogenous rather than exogenous. The rest of the paper is organized as follows: Section 2 provides basic information on the MSM model, while Section 3 presents the data and the empirical results, with Section 4 concluding the paper.

2 Multifractal Models

In this section, we provide a brief description of the mutltifractal model utilized in our volatility forecasting exercises. Multifractal process, which was original introduced in stochastic physics when modelling turbulent dissipation. Analogously, financial markets display properties in common with fluid turbulence. Mandelbrot et al. (1997) first introduced the multifractal apparatus into finance, adapting the approach of Mandelbrot (1974) to an asset-pricing framework, followed by Harte (2001). This multifractal model of asset returns (MMAR) assumes that asset returns r_t follow a compound process, in which an incremental fractional Brownian motion is subordinate to the cumulative distribution function of a multifractal measure. However, the practical applicability of MMAR suffers from the non-causal nature of the time transformation and non-stationarity due to the inherent restriction to a bounded interval. These limitations have been overcome by the development of an iterative version of the multifractal models, including the Markov-switching multifractal model (MSM), cf. Calvet and Fisher (2004) and Lux (2008), which have demonstrated the attractive stochastic properties, with good description of stylized facts in financial markets. In this approach, asset returns r_t are modeled as

$$r_t = \sigma \left(\prod_{i=1}^{k} M_t^{(i)} \right)^{1/2} \cdot \epsilon_t, \quad (1)$$

where σ is the constant scale parameter, and instantaneous volatility is determined by the product of k volatility components or multipliers, $M_t^{(1)}$, $M_t^{(2)}$ ..., $M_t^{(k)}$, with u_t drawn from a standard Normal distribution $N(0,1)$. Each volatility component is renewed at time t with probability γ_i, depending on its rank within the hierarchy of multipliers, or remains unchanged with probability $1 - \gamma_i$. Calvet and Fisher (2004) propose to specify transition probabilities as

$$\gamma_i = 1 - (1 - \gamma_1)^{(b^{i-1})}, \tag{2}$$

with parameters $\gamma_1 \in (0, 1)$ and $b \in (1, \infty)$; while Lux (2008) assumes $\gamma_i = 2^{(k-i)}$. Both specifications guarantee convergence of the discrete-time multifractal process to a limiting continuous-time version with random renewals of the multipliers.

This rather parsimonious approach allows us to preserve the hierarchical structure of MMAR while dispensing with its restriction to a bounded interval. While this model is asymptotically "well-behaved" (i.e. it shares all the convenient properties of Markov-switching processes), it is still capable of capturing several important properties of financial time series including volatility clustering and the power-law behavior of the autocovariance function of absolute moments, cf. Calvet and Fisher (2004) for a detailed proof.

In order to study the interactions and comovements among financial assets, multifractal models can be easily extended to a multivariate setting without imposing too many restrictions such as a bivariate specification. For two financial return series $r_{n,t}$ (for $n = 1, 2$) and assuming that instantaneous volatility is composed of heterogeneous frequencies, the bivariate model of asset returns r_t can be specified as

$$r_t = \sigma \odot [g(M_t)]^{1/2} \odot u_t, \tag{3}$$

where, r_t, σ, and u_t are all bivariate vectors: $r_t = (r_{1,t}, r_{2,t})'$, $\sigma = (\sigma_1, \sigma_2)'$, $u_t = (u_{1,t}, u_{2,t})'$. σ is the vector of constant scale parameters (the unconditional standard deviation); u_t is a 2×1 vector whose elements follow a bivariate standard Normal distribution with an unknown correlation parameter ρ, and $g(M_t)$ is the vector of the products of multifractal volatility components, i.e.

$$g(M_t) = \begin{bmatrix} g(M_{1,t}) \\ g(M_{2,t}) \end{bmatrix}, \tag{4}$$

where each $g(M_{q,t})$ is defined, as in the univariate case, as the product of the volatility components for n series

$$g(M_{n,t}) = \prod_{i=1}^{k} M_{n,t}^{(i)}, \tag{5}$$

with $M_{n,t}^{(i)}$ denoting the volatility component at frequency i of series n. $M_t^{(i)} = (M_{1,t}^{(i)}, M_{2,t}^{(i)})'$. In this specification, $M_t^{(i)}$ are drawn from a bivariate Binomial distribution $M = (M_1,\ \ M_2)'$, with M_1 taking values $m_1 \in (1, 2)$ and $2 - m_1$, and M_2 taking values $m_2 \in (1, 2)$ and $2 - m_2$. Finally, whether or not certain volatility components (new arrivals) are updated for the individual multifractal processes is governed by the transition probabilities γ_i, which are specified as in the univariate version, cf. Eq. (2). The correlation of arrivals between the two series is characterized by a parameter $\lambda \in [0, 1]$, i.e., the probability of a new arrival at hierarchy level i for one time series given a new arrival in the other time series is $(1 - \lambda)\gamma_i + \lambda$. New arrivals are independent if $\lambda = 0$ and simultaneous if $\lambda = 1$.

3 Data and Results

As indicated earlier, we use AAII Sentiment Survey, which is a widely cited measure of sentiment that collects investors' opinion every week . Every Wednesday, editors of Investors Intelligence report the percentage of bullish, bearish, or neutral investors, based on the previous Friday's newsletters' recommendations, with the investor sentiment being measured as the difference between the bullish and bearish percentages. The data is publicly available for download from: www.aaii.com/sentimentsurvey/sent-results. We also use the weekly log-returns of six major stock indices, namely, CRSP, NYSE, AMEX, NASDAQ, S&P500 and DJIA from Datastream as proxies for the overall performance of the stock market from 24/07/1987 to 04/04/2019. Figure 1 presents the plots of empirical data of investors' sentiments (bull-bear spread) and the six stock indices returns. Table 1 reports their pertinent descriptive statistics, and the bottom two rows of heteroskedasticity test and ADF test, which confirm the ARCH effects and stationarity. We study the out-of-sample volatility forecasts assessment and conduct comparisons of univariate and bivariate MSM, and DCC-GARCH models with the in-sample estimation period being 24/07/1987 to 15/04/2004, and the rest of the data being used for the out-of-sample comparison of volatility forecasts.

For the univariate MSM model estimates, we use the simulation based maximum likelihood approach proposed by Calvet et al. (2006). For bivariate MSM models, we estimate the model with pair data, i.e., the bull-bear spread with each of the six stock market indices, respectively. We adopt the same approach with a two-stage procedure, which combines a maximum likelihood estimator for the first group of parameters $\{m_{1,i}, \sigma_i\}$ with $i = \{1,\ 2\}$, $i = 1$ refers to sentiment data, and $i = 2$ refers to stock markets indices, i.e., CRSP; DJIA; NYSE; SP500; NASDAQ; AMEX. For the second stage, we keep the first set of estimates, and estimate the second group $\{\rho,\ \lambda\}$ via simulation based maximum likelihood estimation which is implemented via particle filter. The two-stage approach allows to reduce computation time compared to the full maximum likelihood approach, it also makes the choice of larger number of cascade level k feasible, and we use $k = 8$ which is consistent to existing literature. The first four of these parameters could be identified by estimates for a univariate multifractal model, while the remaining ones require the complete bivariate data set.

Table 2 reports the in-sample univariate and bivariate multifractal model estimates. The estimates for sentiments are reported in the second column, with subsequent columns reporting the estimates including the bivariate parameters estimates of ρ and λ for stock indices. In terms of fractality of volatility as measured by the parameters m_1 (note: $m_1 \in [1, 2]$), we observe sound fractality and with not much fluctuations across all data. For the unconditional volatility parameters σ_i, estimate of Bull-Bear Spread shows apparently much higher volatility comparing with ones of stock indices; among the six stock indices, AMEX show relatively high volatility level. In terms of the correlation of innovations, ρ, all data exhibit positive though somewhat weak correlations between sentiment and stock market index. We also observe positive correlation across the sentiment and stock market indices often pertains to volatility arrivals λ, which appear quite plausible.

Table 3 reports in sample estimates of DCC-GARCH(1,1) model of Engle (2002). DCC-GARCH model is an extension of the conventional GARCH (1,1) model of Engle (1982), specified as $r = \mu + \epsilon_t$, and $\epsilon_t | I_{t-1} \sim N(0, \sigma_t)$, with the volatility process following $\sigma_t = \omega + \alpha \cdot \epsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$. The GARCH parameters for each series$(\omega, \beta, \alpha, \mu)$ are with their usual interpretation. In the bottom panel of Table 3 presents the empirical estimates for the DCC-GARCH, which has a non-linear GARCH type specification for the conditional correlation: $Q_t = (1 - a - b)\bar{Q} + a\epsilon_t \cdot \epsilon'_{t-1} + bQ_{t-1}$. a and b are the so-called news and decay coefficients, respectively. $\bar{Q} = E[\epsilon_t \cdot \epsilon'_{t-1}]$ is the unconditional variance matrix of the standardized residuals (the unconditional correlation) and ρ_{12} represents the unconditional correlation coefficient in matrix $\bar{Q}$.

Table 4 presents the performance metrics for out-of-sample forecasts from univariate and bivariate multifractal models, DCC-GARCH(1, 1) model respectively. We consider various forecast horizons ranging from 1 to 100 steps. We report in the tables the relative mean square error (MSE) and the relative mean absolute error (MAE), computed by dividing the MSE and MAE estimates by the pertinent MSE and MAE of the naive volatility predictor (using historical volatility), therefore any values smaller than 1 indicate an improvement against historical volatility.

Let us first look at the forecast performances based on the univariate and bivariate MSM models whose MSE and MAE are reported in 4th and 5th columns of Table 4. It is not surprising to observe that there are most of forecasts are superior to ones based on historical volatility as can be seen most of values <1. By comparing with levels of MSE and MAE of univariate and bivariate MSM models, one can find results based on bivariate MSM models almost unanimously generate smaller values than univariate models. By conducting the Diebold-Mariano test (cf. Diebold and Mariano (1995)) reported in the 7th column of table 4 labeled p(DM), it appears there are only limited significance cases and most of them are at at 10% level, namely one case of CRSP at 1-week horizon for MSE, and 3 cases for MAE; one case of DJIA for MAE; for NYSE, there are 2 cases for each MSE and MAE criteria respectively; there are also 3 MAE cases for NASDAQ at 10% level significance, as well as MAE cases of AMEX which are close to 5% level. Though there are many cases too small to be significant under the Diebold-Mariano tests, we on the other hand, performed forecast encompassing tests using the regression (cf. Harvey et al., 1998):

$$e_{1,t} = \theta(e_{1,t} - e_{2,t}) + \epsilon_t \quad (6)$$

with $e_{1,t}$ and $e_{2,t}$ being the errors of the forecasts from univariate and bivariate MSM models. The encompassing test tests the null hypothesis of $H_0 : \theta = 0$ which means that the univariate MSM model encompasses the bivariate setting, rejection of H_0, i.e. estimates of θ significantly different from zero indicates that the bivariate MSM model contributes useful information to the forecast problem in question on top of what the univariate model already contributes. The 8th and 9th columns of Table 4 provide the estimates and standard error of θ. Our results are not entirely homogeneous, but are quite encouraging. We find majority estimates of θ significantly different from zero for various horizons, except with one case for CRSP, NYSE and SP500; and two cases for DJIA and AMEX. These results imply that univariate MSM model does not encompass the forecasts from the bivariate models at most forecast horizons. Hence, the bivariate MSM models do add significant value on top of forecasts based on univariate MSM model.

Next, we turn our attention to compare with the DCC-GARCH model forecasting. The 6th column of Table 4 reports the relative MSE and MAE values for the volatility forecasts based on the bivariate DCC-GARCH model. We have also conducted the Diebold-Mariano test whose results are reported in the last column of Table 4 labeled as p(DM). Apparently bivariate multifractal models outperform the DCC-GARCH model in most cases according to both MSE and MAE criteria across all stock indices (62 out of total 72 cases). The DCC- GARCH model generally produces better forecasts in most short-term horizons, i.e. one case of CRSP at 1-week horizon for MSE; 2 cases of DJIA at 1 and 5-week horizons for MSE; one MSE and one MAE cases at 1-week horizon for S&P500 and NASDAQ respectively; for AMEX, there are 2 cases for MSE at 1 and 5-week horizons and one MAE case at 1-week horizon. It is actually plausible that multifractal model offers more accurate predictions at all longer horizons as it genuinely captures long memory of volatility, also be noted, MSM models are parsimonious with much less number of parameters comparing with multivariate GARCH models.

4 Concluding remarks

In this paper, we study the role of investors' uncertainty, as measured by the conditional volatility of investor sentiment, in forecasting volatility of six major stock market indices using bivariate Markov-switching multifractal (MSM) models. Our results demonstrate that forecasting stock markets volatility in combination with information associated with investors' uncertainty generates smaller forecasts errors, relative to univariate MSM models, which does not include the metric of uncertainty. We also find that forecasts based on bivariate MSM are superior to the DCC-GARCH models, in particular at longer horizons. Our results suggest that investors can improve their portfolio allocation and risk management by accommodating the role of uncertainty into their models of volatility.

As part of future research, it would be interesting to extend our analysis to other developed and emerging markets, and even analyze the importance of US investors' uncertainty for cross-market volatility.

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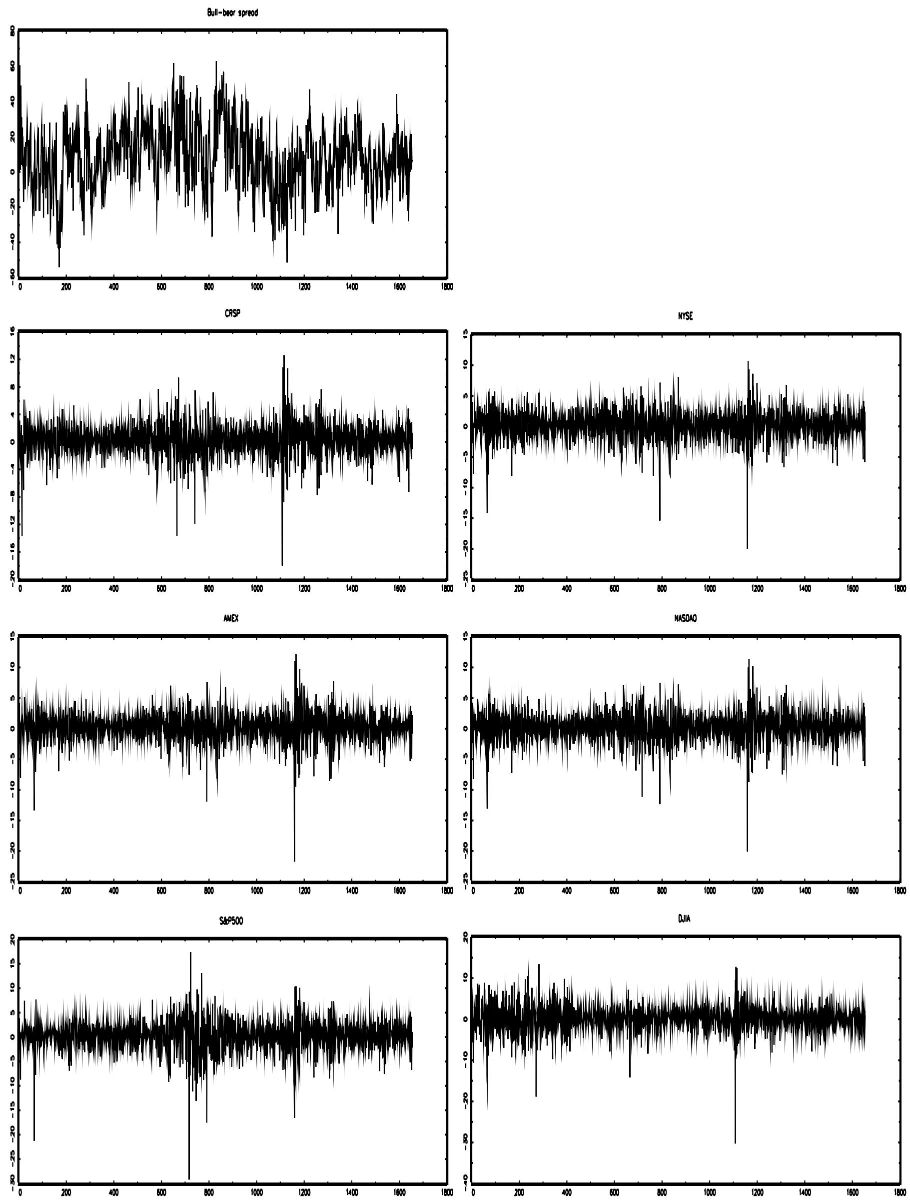

Figure 1: Investors' opinion and stock indices returns

Note: The first figure is investors opinion (sentiment) measured by bull-bear spread, the below 6 figures are returns of the stock indices: CRSP (value-weighted returns from Center for Research in Security Prices), NYSE (New York Stock Exchange), AMEX (American Stock Exchange), NASDAQ (National Association of Securities Dealers Automated Quotations), S&P500, and DJIA (Dow Jones Industrial Average), respectively.

Table 1: Selected descriptive statistics of the data.

	Spread	CRSP	DJIA	NYSE	SP500	NASDAQ	AMEX
mean	7.881	0.204	0.156	0.129	0.144	0.175	0.097
Std.dev	17.689	2.298	2.264	2.252	2.283	2.994	2.921
Min	-54	-17.98	-20.03	-21.735	-20.084	-29.175	-30.298
Max	62.9	12.617	10.698	12.128	11.356	17.377	13.353
Skewness	-0.039	-0.701	-0.969	-1.04	-0.869	-1.154	-1.039
Kurtosis	3.085	9.051	10.13	11.83	9.706	12.89	13.268
ARCH	86.51(0.00)	233.06(0.00)	125.18(0.00)	228.61(0.00)	197.57(0.00)	132.24(0.00)	118.87(0.00)
ADF	-9.974[4]	-28.111[1]	-28.103[1]	-27.966[1]	-28.227[1]	-27.438[1]	-27.621[1]

Note: This table reports a range of commonly used descriptive statistics for the bull-bear spread and six U.S stock market indices; namely, CRSP, NYSE, AMEX, NASDAQ, SP500, DJIA. The specific statistics reported are the mean price, its standard deviation, minimum (min.) and maximum (max.) prices, skewness and kurtosis of price, followed by heteroskedasticity test: we filter the price series using an autoregressive model with an order of 12 and then implement the Lagrange-Multiplier test examining the null-hypothesis of no ARCH. The LM test statistic and the p-value (in parenthesis) are reported. The last row contains an Augmented Dickey and Fuller (DF, 1981) unit root test. The optimal lag length in the ADF regression is chosen using the Schwarz Information criterion (SIC); we begin with a maximum of eight lags and then use the SIC to select the optimal lag length. The optimal lag lengths are reported in square brackets beside the ADF test statistic.

Table 2: In-sample estimates of univariate and bivariate MSM models.

	Spread	CRSP	DJIA	NYSE	SP500	NASDAQ	AMEX
m_1	1.225	1.251	1.262	1.263	1.254	1.306	1.285
	(0.026)	(0.025)	(0.028)	(0.028)	(0.026)	(0.031)	(0.026)
σ	22.451	2.242	2.337	2.082	2.203	2.330	3.067
	(2.102)	(0.273)	(0.252)	(0.227)	(0.266)	(0.227)	(0.250)
ρ		0.097	0.126	0.117	0.024	0.038	0.132
		(0.022)	(0.013)	(0.010)	(0.010)	(0.013)	(0.015)
λ		0.150	0.135	0.151	0.135	0.146	0.128
		(0.018)	(0.019)	(0.012)	(0.012)	(0.026)	(0.017)

Note: This table reports the in-sample estimates of the univariate and bivariate MSM models for invertors' sentiment, and six major stock indices, namely, CRSP, DJIA, NYSE,S&P500, NASDAQ and AMEX. the first two rows are parameters estimates of $\{m_1, \sigma\}$ for univariate MSM models. Since we adopt two-stage estimation for bivariate MSM models via particle filter apprach (with number of particles being 10000), the estimates of $\{m_1, \sigma\}$ for each individual time series data are almost identical with univariate models, we skip to report them in pairs again to save spaces, while reporting the extra bivariate parameters estimates of $\{\rho, \lambda\}$ in the bottom two rows, numbers in parenthesis are standard errors.

Table 3: In-sample estimates of bivariate DCC-GARCH (1, 1)models.

	Spread	CRSP	DJIA	NYSE	SP500	NASDAQ	AMEX
μ	11.923	0.305	0.269	0.228	0.220	0.369	0.202
	(0.862)	(0.061)	(0.070)	(0.060)	(0.065)	(0.085)	(0.086)
ω	52.508	0.103	0.189	0.148	0.063	0.261	0.064
	(25.534)	(0.083)	(0.324)	(0.175)	(0.066)	(0.137)	(0.057)
α	0.341	0.145	0.139	0.158	0.108	0.251	0.076
	(0.090)	(0.072)	(0.102)	(0.088)	(0.065)	(0.101)	(0.031)
β	0.515	0.845	0.835	0.820	0.886	0.752	0.920
	(0.155)	(0.071)	(0.147)	(0.110)	(0.066)	(0.081)	(0.032)
a		0.008	0.000	0.000	0.000	0.000	0.006
		(0.004)	(0.000)	(0.000)	(0.000)	(0.000)	(0.015)
b		0.987	0.845	0.848	0.842	0.824	0.962
		(0.008)	(0.339)	(1.291)	(0.346)	(2.570)	(0.127)
ρ_{12}		0.077	0.039	0.021	0.032	0.052	0.129
		(0.097)	(0.034)	(0.035)	(0.034)	(0.034)	(0.039)
lkl		-5561.964	-5632.855	-5530.574	-5596.696	-5804.078	-5862.729

Note: This table reports the in-sample estimates of the DCC-GARCH(1, 1) models for invertors' sentiment, and six major stock indices, namely, CRSP, DJIA, NYSE,S&P500, NASDAQ and AMEX, numbers in parenthesis are standard errors. the last row reports their maximized log likelihood values

Table 4: Comparison of volatility forecasts

Data			Uni MSM	Biv MSM	DCC-GARCH	Uni vs Biv MSM			Biv MSM vs. DCC-GARCH
CRSP	Horizons	MSE				p(DM)	θ	std.er	p(DM)
	1		0.898	0.88	0.875	0.104	0.496	0.088	0.339
	5		0.961	0.946	0.967	0.317	0.387	0.092	0.012
	10		0.989	0.983	1.07	0.713	0.155	0.071	0.009
	20		0.991	0.983	1.06	0.614	0.207	0.104	0.025
	50		1.015	1.011	1.094	0.723	-0.362	0.165	0.011
	100		1.051	1.056	1.09	0.626	-0.061	0.257	0.048
		MAE							
	1		0.886	0.866	0.913	0.097			0.053
	5		0.915	0.888	0.989	0.085			0.006
	10		0.93	0.912	1.063	0.102			0.005
	20		0.954	0.94	1.13	0.209			0.005
	50		0.976	0.965	1.286	0.207			0.004
	100		1.029	1.028	1.421	0.921			0.003
DJIA	Horizons	MSE				p(DM)	θ	std.er	p(DM)
	1		0.929	0.917	0.945	0.278	0.351	0.118	0.054
	5		0.973	0.963	0.997	0.536	0.297	0.136	0.036
	10		0.984	0.979	1.017	0.804	0.147	0.017	0.039
	20		0.988	0.982	1.014	0.543	-0.225	0.206	0.079
	50		1.016	1.018	1.052	0.793	0.676	0.497	0.061
	100		1.044	1.054	1.016	0.657	-1.382	0.271	0.085
		MAE							
	1		0.829	0.821	0.847	0.456			0.096
	5		0.853	0.845	0.911	0.375			0.051
	10		0.873	0.875	0.969	0.802			0.041
	20		0.896	0.9	1.033	0.621			0.016
	50		0.934	0.947	1.156	0.194			0.013
	100		0.978	1.001	1.188	0.079			0.017
NYSE	Horizons	MSE				p(DM)	θ	std.er	p(DM)
	1		0.923	0.897	0.9	0.094	0.516	0.085	0.576
	5		0.973	0.952	0.961	0.102	0.428	0.094	0.309
	10		0.992	0.981	1.048	0.256	0.396	0.109	0.074
	20		0.99	0.98	1.013	0.253	0.406	0.144	0.089
	50		1.003	0.999	1.021	0.699	-0.399	0.325	0.081
	100		1.032	1.036	1.021	0.724	1.83	0.431	0.097
		MAE							
	1		0.915	0.914	0.937	0.881			0.104
	5		0.936	0.934	1.002	0.797			0.057
	10		0.959	0.964	1.083	0.306			0.042
	20		0.985	0.997	1.138	0.122			0.029
	50		1.005	1.028	1.213	0.103			0.035
	100		1.053	1.087	1.232	0.068			0.084
SP500	Horizons	MSE				p(DM)	θ	std.er	p(DM)
	1		0.911	0.899	0.916	0.115	0.401	0.098	0.206
	5		0.967	0.954	0.98	0.172	0.36	0.099	0.094
	10		0.989	0.982	1.061	0.226	0.382	0.104	0.055
	20		0.988	0.982	1.065	0.374	0.207	0.131	0.029
	50		1.013	1.011	1.128	0.722	-0.298	0.133	0.011
	100		1.046	1.051	1.152	0.239	-0.425	0.173	0.021
		MAE							
	1		0.863	0.85	0.871	0.166			0.153
	5		0.89	0.874	0.935	0.108			0.042
	10		0.907	0.897	1.016	0.238			0.035
	20		0.929	0.924	1.095	0.284			0.018
	50		0.963	0.964	1.287	0.903			0.005
	100		1.017	1.027	1.492	0.126			0.002
NASDAQ	Horizons	MSE				p(DM)	θ	std.er	p(DM)
	1		0.798	0.794	0.775	0.238	0.215	0.092	0.224
	5		0.894	0.883	0.993	0.269	-0.268	0.093	0.085
	10		0.959	0.953	1.165	0.462	-0.208	0.097	0.033
	20		0.97	0.959	1.23	0.201	-0.214	0.099	0.025
	50		1.028	1.018	1.793	0.181	0.422	0.094	0.003
	100		1.098	1.086	1.825	0.235	0.209	0.064	0.004
		MAE							
	1		0.732	0.719	0.707	0.238			0.248
	5		0.763	0.747	0.806	0.097			0.083
	10		0.787	0.775	0.924	0.131			0.042
	20		0.809	0.791	1.1	0.112			0.018
	50		0.854	0.837	1.701	0.104			0.007
	100		0.912	0.894	1.788	0.092			0.002
AMEX	Horizons	MSE				p(DM)	θ	std.er	p(DM)
	1		0.938	0.937	0.956	0.948	0.199	0.068	0.136
	5		0.976	0.969	1.011	0.752	-0.338	0.185	0.107
	10		1.015	1.001	1.066	0.216	-0.375	0.197	0.048
	20		1.017	1.003	1.069	0.198	-0.454	0.205	0.054
	50		1.028	1.012	1.067	0.169	0.585	0.243	0.067
	100		1.038	1.022	1.063	0.116	0.715	0.315	0.071
		MAE							
	1		0.704	0.679	0.702	0.058			0.111
	5		0.743	0.704	0.778	0.043			0.044
	10		0.776	0.733	0.852	0.051			0.025
	20		0.798	0.75	0.886	0.045			0.036
	50		0.838	0.786	0.977	0.039			0.022
	100		0.877	0.823	1.086	0.043			0.014

Note: The table exhibits mean squared errors (MSE) and mean absolute errors (MAE) of out-of-sample volatility forecasts for univariate and bivariate MSM models and DCC-GARCH models. MSE and MAE as reported in the table have been standardized by dividing by the MSE and MAE of a naive forecast using historical volatility (so that values <1 indicate an improvement against historical volatility), p(DM) denotes the probability of the Diebold-Mariano test, while θ is the slope estimate of the forecast encompassing regression, Eq. (6), followed by its standard error in the subsequent column.