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## **Is the Response of the Bank of England to Exchange Rate Movements Frequency-Dependent?**

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# Is the Response of the Bank of England to Exchange Rate Movements Frequency-Dependent?

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## Abstract

In this paper, we estimate a Small Open Economy Dynamic Stochastic General Equilibrium (SOEDSGE) model of the United Kingdom (UK), with the main focus being to test the hypothesis whether the Bank of England (BoE) responds to (frequency-dependent) exchange rate movements or not. For our purpose, we use an extended quarterly data set spanning the period of 1986:Q1 to 2018:Q1, which in turn includes the zero lower bound situation, and also estimate the SOEDSGE model based on observable data decomposed into its frequency components, under the presumption that central banks is more comfortable in responding to long-term fundamental movements in exchange rates. We find that the BoE not only responds to exchange rate movements in a statistically significant manner, but also

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that it primarily focuses on long-term movements of currency depreciations more strongly than short-term fluctuations of the same.

*Keywords:* Small Open Economy DSGE Model, Monetary Policy Rule, Exchange Rate, Structural Estimation, Bayesian Analysis, Wavelets

*JEL Classification:* C32, E52, F41.

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## 1. Introduction

The United Kingdom (UK), just like the United States (US), is a major player in the world economy, with monetary policy decisions of the Bank of England (BoE) being of interest to both academicians and financial markets. Given this, what variables determine the interest rate-setting behaviour of the BoE is, understandably, an important question. While the role of the output-gap and inflation rate in determining the policy rate of the BoE, and central banks across the world, is well-accepted along the lines of the Taylor-rule (Taylor (1993)), whether information in exchange rate movements should also be accounted for remains a debatable issue.

UK is a natural resource exporter, and hence, domestic business cycle fluctuations are likely to have substantial international relative price components. In addition, monetary policy is partly transmitted to the real economy through its effect on the exchange rate. The BoE therefore may have a specific interest in explicitly reacting to and smoothing exchange rate movements as a predictor of domestic volatility. However, based on various alternative econometric approaches (for example, single-equation interest rate rules, structural vector autoregressions (SVARs), Small Open Economy Dynamic Stochastic General Equilibrium (SOEDSGE) models), evidence regarding that the BoE responds to (nominal) exchange rate movements is mixed (see

for example, [Lubik and Schorfheide \(2007\)](#), [Dong \(2013\)](#), [Bjornland and Halvorsen \(2014\)](#)). <sup>1</sup>

Low frequency movements in exchange rates are likely to be tied with fundamentals more than high-frequency movements of the same, which in turn could be associated with speculation, and hence (harder to predict) random behaviour ([Rapach and Wohar \(2002\)](#), [Balke, Ma, and Wohar \(2013\)](#), [Caraiani \(2017\)](#)). Given this, it is possible that central bankers find it more comfortable to respond to long-term (i.e., low-frequency) movements of the exchange rate rather than its corresponding short-term fluctuations. With this hypothesis in mind, the objective of this paper is to revisit the question of whether the BoE respond to exchange rate movements, with us now analyzing not only the aggregate nominal effective exchange rate depreciations, but also its various frequency components. Given the well-known econometric issues associated with single-equation rule-type and atheoretical VAR approaches in light of the Lucas Critique ([Lucas \(1976\)](#)), we estimate the SOEDSGE model of [Lubik and Schorfheide \(2007\)](#) for the UK to provide an answer to our question, over the period of 1986:Q1 to 2018:Q1. While, closed-economy frequency-based models for the US economy has been estimated before (see, [Caraiani \(2015\)](#) for a detailed discussion in this regard), to the best of our knowledge, this is the first attempt to estimate a SOEDSGE model in both time and frequency-domains to determine whether the BOE's response to exchange rate movements is contingent on its frequency components.

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<sup>1</sup>But, some evidence in favor of the fact that the BoE does weigh in exchange rate movements in its interest rate decisions can be observed when one allows for structural regime-switches (see for example, [Chen and Macdonald \(2012\)](#), and [Alstadheim, Bjornland, and Maih \(2013\)](#)).

The remainder of the paper is organized as follows: Section 2 lays out the basics of the SOEDSGE and the frequency decomposition of the data using wavelet, Section 3 presents the data and results, with Section 4 concluding the paper.

## 2. Theoretical and Empirical Frameworks

In this section, we introduce the open-economy DSGE model used in the empirical analysis, and detail the wavelet filtering method used to decompose the observable data series used in the estimation of the DSGE model.

### 2.1. An Open Economy DSGE Model

The model we use is one of the reference models used in the past to answer to the question whether the central banks react or not to exchange rates, [Lubik and Schorfheide \(2007\)](#). The model is a simplified version of the reference model in [Gali and Monacelli \(2005\)](#).

$$y_t = E_t y_{t+1} - (\tau + \alpha (2 - \alpha) (1 - \tau)) (R_t - E_t \pi_{t+1}) - \rho_z z_t - \alpha (\tau + \alpha (2 - \alpha) (1 - \tau)) E_t \Delta q_{t+1} + \alpha (2 - \alpha) \frac{1 - \tau}{\tau} E_t \Delta y_{t+1}^* \quad (1)$$

The first equation is an open economy IS curve. Here,  $y_t$  is the output,  $R_t$  the nominal interest rate,  $\pi_t$  the domestic inflation,  $q_t$  the terms of trade and  $y_t^*$  the foreign output. The parameter  $\alpha$  is the import share, while  $\tau$  is the intertemporal substitution elasticity.

$$\pi_t = \beta E_t \pi_{t+1} + \alpha \beta E_t \Delta q_{t+1} - \alpha \Delta q_t + \frac{\kappa}{\tau + \alpha (2 - \alpha) (1 - \tau)} (y_t - \bar{y}_t) \quad (2)$$

Equation 2 is the open economy economy equivalent of a New Keynesian Phillips curve. Here  $\bar{y}$  is the potential output, i.e. that level of output when prices nominal rigidities are missing. The parameter  $\kappa$  is determined by factors like labor supply and demand elasticities or by price stickiness. The potential output is defined below, in equation 3.

$$\bar{y}_t = -\alpha \frac{(1 - \tau) (2 - \alpha)}{\tau} y_t^* \quad (3)$$

$$\pi_t = \Delta e_t + \Delta q_t (1 - \alpha) + \pi_t^* \quad (4)$$

Equation 4 above shows the link between domestic inflation  $\pi_t$ , nominal exchange rate  $e_t$ , terms of trade  $q_t$  as well as foreign inflation, denoted by  $\pi_t^*$ .

$$R_t = \rho_r R_{t-1} + (1 - \rho_r) (\psi_1 \pi_t + \psi_2 y_t + \psi_3 \Delta e_t) + \epsilon_{r3t} \quad (5)$$

The last equation, equation 5, introduces a standard Taylor, modified to include the reaction to exchange rate movements. The parameter  $\rho_r$  characterizes the degree of interest rate smoothing, while  $\psi_1, \psi_2, \psi_3$  correspond to inflation, output and exchange rate movements.

$$\Delta q_t = \rho_q \Delta q_{t-1} + \epsilon_{qt} \quad (6)$$

Following the original paper, [Lubik and Schorfheide \(2007\)](#), changes in terms of trade are assumed to follow an AR(1) process, equation 6. Finally, AR(1) processes are also assumed for world technology,  $z_t$ , foreign output  $y_t^*$  and foreign inflation  $\pi_t^*$ , see equations 7-9.

$$z_t = \rho_z z_{t-1} + \epsilon_{z_t} \quad (7)$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \epsilon_{y_t^*} \quad (8)$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \epsilon_{\pi_t^*} \quad (9)$$

## 2.2. Wavelet Decomposition

A known issue in estimating DSGE model is that the filtering of the series must not use forward information, but only the backward observations to derive the current filtered value. This is a known issue for several filtering methods, including the double-sided Hodrick-Prescott filter. In the context of wavelets, the standard filtering using wavelets suffers from the same deficiency. To address this shortcoming, we use the redundant wavelet transform, see [Aussem, Campbell, and Murtagh \(1998\)](#) and [Zheng, Starck, Campbell, and Murtagh \(1999\)](#). This approach has already been used in [Caraiani \(2017\)](#) to forecast exchange rate on different frequencies.

The approach we use is based on the redundant Haar Wavelet Transform. The main advantage being that it performs the time-scale decomposition using only the previous data-points. Below, we present the algorithm for general redundant discrete wavelet transform, which is also known as *the à trous wavelet transform*.

We start from a series  $c_0(k)$ . The initial series is decomposed into wavelets components, as well as a smooth component. At each scale  $j$  the latter is denoted by

$c_j(k)$ . The initial series can be written as the scalar product at samples  $k$  of the function  $f(x)$  and the scaling function  $\phi(x)$ .

$$c_0(k) = \langle f(x), \phi(x - k) \rangle \quad (10)$$

The scalar function is selected such that the following equation holds (also known as the dilation equation)

$$\frac{1}{2}\phi\left(\frac{x}{2}\right) = \sum_l h(l)\phi(x - l) \quad (11)$$

$h$  stands for the low-pass filter, corresponding to  $\phi_x$ . Based on these equations, we can derive the smooth component at resolution  $j$  for any observation  $k$  as follows:

$$c_j(k) = \frac{1}{2^j} \langle f(x), \phi\left(\frac{x - k}{2^j}\right) \rangle \quad (12)$$

For two consecutive resolutions, the difference between them can be denoted by  $w_j$ . Thus, we obtain:

$$w_j(k) = c_{j-1}(k) - c_j(k) \quad (13)$$

This can also be written as:

$$w_j(k) = \frac{1}{2^j} \langle f(x), \psi\left(\frac{x - k}{2^j}\right) \rangle \quad (14)$$

Thus, we obtained the discrete wavelet transform using the *à trous* algorithm.  $\psi$  denotes wavelet function given by:



$$\frac{1}{2}\psi\left(\frac{x}{2}\right) = \phi(x) - \frac{1}{2}\phi\left(\frac{x}{2}\right) \quad (15)$$

Using this algorithm, we can decompose the initial series as the sum of wavelet components  $w_j$  and a smooth component  $c_p$ :

$$c_0(k) = c_p + \sum_{j=1}^p w_j(k) \quad (16)$$

### 3. Empirical Analysis

#### 3.1. Data

The SOEDSGE model is fitted to data on output growth, inflation, nominal interest rates, exchange rate changes, and terms of trade changes. We consider seasonally adjusted quarterly data for the UK covering the period of 1986:Q1 to 2018:Q1. The series were obtained (primarily) from the Main Economic Indicators (MEI) database of the Organisation for Economic Co-operation and Development (OECD). The output series is real GDP in per-capita terms, inflation is computed using the Consumer Price Index. The nominal interest rate is a short-term rate. However, given the zero lower bound situation of the monetary policy instrument in the wake of the “Great Recession”, we use the shadow short rate developed by [Wu and Xia \(2016\)](#), based on its availability, over the period of 1990 (till the end of the sample), and the regular short-term-rate prior to that. Note that, the shadow short rate is the nominal interest rate that would prevail in the absence of its effective lower bound, with it derived by modeling the (three-factors) term structure of the yield curve, and has been shown by [Wu and Xia \(2016\)](#) to be a close approximation of the

short-term rate during the conventional periods of monetary policy decision-making. As nominal exchange rate variable we use a nominal trade-weighted exchange rate index, whereas the terms of trade are measured as the (natural log) ratio of export and import price indices. We de-mean the data prior to estimation.

### 3.2. *Estimating the DSGE Model across Time and Frequency*

We provide estimations for the DSGE model for both the aggregate demeaned series, as well as the wavelet components. Although there is some effort to estimate structural equations along different wavelet components, see Gallegati, Gallegati, Ramsey, and Semmler (2011), or structural DSGE models, see Caraiani (2015), a problem that negatively affected previous work was the fact that the wavelet decomposition did not take into account the forward character of standard wavelet transform. In contrast, in this paper, we use a redundant wavelet transform based on the Haar wavelet that is purely backward looking.

In Table 1, we first provide estimations for the aggregate series. We also provide estimations for the wavelet components,  $W_1$  to  $W_4$  in Tables 2 to 5. Each component  $W_i$  captures the changes in the interval  $[2^i, 2^{i+1}]$ . Thus the  $W_1$  component measures the dynamics between 2 and 4 quarters while the  $W_4$  component does the same for the changes between 16 and 32 quarters (4 to 8 years). Based on the results, we make the following two main observations:

First from Table 6, using the marginal data densities obtained under the models allowing for response of the interest rate to exchange rate movements and then restricting it to zero, we find that the former (unrestricted) model has a better fit than the restricted version of the same, with the results holding for not only the

aggregate data, but also in the cases of the wavelet components ( $W_1$ ,  $W_2$ ,  $W_3$  and  $W_4$ ). In terms of the Bayes factor reported in the same table, we can say that the unrestricted model performs significantly better for the aggregate series, and the wavelet components  $W_2$ ,  $W_3$  and  $W_4$ , with the highest gain obtained under the longest frequency considered ( $W_4$ ), i.e., at changes in exchange rates at between 16 and 32 quarters.<sup>2</sup>

Second, when comparing the results across Tables 1 to 5, we observe that, for some of the parameters, there is a tendency to move in certain patterns across the different frequencies. For example, the autocorrelation coefficients are stronger at detail  $W_4$  as compared to detail  $W_1$ . The coefficient attached to inflation is also weaker at the first detail  $W_1$ , while there is also a tendency toward stronger responses of the interest rate to exchange rate, although for the latter case, this is not verified for  $W_4$ . The results are similar in essence with the findings in the previous related work, like [Caraiani \(2015\)](#) or [Sala \(2015\)](#). It must be noted however, that [Caraiani \(2015\)](#) did not find a clear pattern for the Taylor rule coefficients, but mostly for structural parameters related to the behavior of households and firms in the New Keynesian DSGE model used by the author.

In sum, we can draw two main conclusions: (a) The marginal densities of the DSGE model increases for the frequency-based estimations when compared to the aggregate series, with the fit increasing massively in a consistent manner as the

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<sup>2</sup>Using the original data span of [Lubik and Schorfheide \(2007\)](#) ending in 2003, the biggest gain in terms of the Bayes factor was observed under the aggregate series, with the results available upon request from the authors. Clearly then, over the recent periods, the BoE has started to respond more strongly to the exchange rate depreciations, and in particular to long-frequency movements of the same.

BoE targets lower frequency movements in the exchange rate, and; (b) While, the response of interest rate to exchange rate movements is the strongest under the aggregate series (which makes sense given that the original data series is the sum of all four frequency components),<sup>3</sup> the explanatory power of the structural model to explain the behavior of the economy of the UK, especially in terms of the BoE responding to exchange rate depreciations, particularly at longer horizons, is much higher than not responding at all.

#### 4. Conclusions

The existing literature provides mixed evidence in terms of whether the BoE responds to exchange rate movements or not. Given this, we revisit this question, but now we analyze not only aggregate nominal effective exchange rate depreciations, but also its various frequency components, with the belief that low- frequency movements in exchange rates are likely to be tied with fundamentals more than high-frequency movements of the same, and hence, central bankers might find it more comfortable to respond to such long-term fluctuations. We estimate a SOEDSGE model to provide an answer to our question, over the extended period (relative to existing studies) of 1986:Q1 to 2018:Q1, which in turn, also include the zero lower bound period of the interest rates. Unlike the conflicting evidence in existing studies, we find

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<sup>3</sup>This observation, though statistically insignificant as in Bjornland and Halvorsen (2014), is also verified when we compare the impulse response of the interest rate to an exchange rate shock for the aggregate and wavelet-decomposed data series, obtained from a VAR model, whereby the shock is identified using a Choleski decomposition (i.e., with a recursive ordering of output growth, inflation, terms of trade changes, nominal interest rates, and exchange rate changes). Complete details of these results are available upon request from the authors.

evidence that the BoE not only responds to exchange rate movements in a statistically significant manner (likely driven by the extended sample), but also the fact that it primarily focuses on long-term movements of currency depreciations more strongly than short-term fluctuations of the same.

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Table 1: Results from Metropolis-Hastings for Aggregate Series

		Prior		Posterior			
		Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf HPD sup
$\kappa$	gamma	0.500	0.2500	0.474	0.1046	0.3078	0.6327
$\psi_1$	gamma	1.500	0.5000	3.120	0.3418	2.5576	3.6670
$\psi_2$	gamma	0.250	0.1500	0.095	0.0273	0.0492	0.1390
$\psi_3$	gamma	0.250	0.1500	0.237	0.0601	0.1380	0.3350
$\tau$	beta	0.500	0.2000	0.239	0.0448	0.1666	0.3074
$\rho_r$	beta	0.500	0.1000	0.801	0.0292	0.7560	0.8496
$\rho_q$	beta	0.400	0.2000	0.060	0.0386	0.0031	0.1139
$\rho_{\pi^*}$	beta	0.800	0.1000	0.426	0.0648	0.3197	0.5327
$\rho_{y^*}$	beta	0.900	0.1000	0.996	0.0033	0.9918	1.0000
$\rho_z$	beta	0.200	0.0500	0.582	0.0073	0.5729	0.5897
$\epsilon_r$	invg	0.500	4.0000	0.167	0.0205	0.1339	0.1991
$\epsilon_q$	invg	1.500	4.0000	0.543	0.0344	0.4862	0.5980
$\epsilon_{y^*}$	invg	1.500	4.0000	0.461	0.1151	0.2900	0.6317
$\epsilon_{\pi^*}$	invg	0.500	4.0000	1.402	0.0892	1.2563	1.5461
$\epsilon_z$	invg	1.000	4.0000	0.385	0.0458	0.3087	0.4553





Table 2: Results from Metropolis-Hastings for W1 Component

	Dist.	Prior		Posterior			
		Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
$\kappa$	gamma	0.500	0.2500	2.804	0.4439	2.0656	3.5245
$\psi_1$	gamma	1.500	0.5000	1.926	0.4362	1.1912	2.6131
$\psi_2$	gamma	0.250	0.1500	0.666	0.3300	0.1516	1.2275
$\psi_3$	gamma	0.250	0.1500	0.096	0.0341	0.0401	0.1499
$\tau$	beta	0.500	0.2000	0.652	0.0668	0.5472	0.7676
$\rho_r$	beta	0.500	0.1000	0.430	0.0694	0.3166	0.5439
$\rho_q$	beta	0.400	0.2000	0.025	0.0180	0.0012	0.0492
$\rho_{\pi^*}$	beta	0.800	0.1000	0.135	0.0256	0.1025	0.1703
$\rho_{y^*}$	beta	0.900	0.1000	0.125	0.0426	0.0551	0.1885
$\rho_z$	beta	0.200	0.0500	0.117	0.0267	0.0755	0.1618
$\epsilon_r$	invg	0.500	4.0000	0.131	0.0194	0.1005	0.1612
$\epsilon_q$	invg	1.500	4.0000	0.394	0.0250	0.3522	0.4336
$\epsilon_{y^*}$	invg	1.500	4.0000	0.643	0.2161	0.3406	0.9760
$\epsilon_{\pi^*}$	invg	0.500	4.0000	0.751	0.0497	0.6696	0.8298
$\epsilon_z$	invg	1.000	4.0000	0.544	0.1379	0.3275	0.7702

Table 3: Results from Metropolis-Hastings for W2 Component

	Dist.	Prior		Posterior			
		Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
$\kappa$	gamma	0.500	0.2500	1.429	0.2410	1.0274	1.7964
$\psi_1$	gamma	1.500	0.5000	2.751	0.4167	2.0531	3.3984
$\psi_2$	gamma	0.250	0.1500	0.185	0.0992	0.0300	0.3305
$\psi_3$	gamma	0.250	0.1500	0.161	0.0437	0.0902	0.2321
$\tau$	beta	0.500	0.2000	0.713	0.0674	0.6068	0.8246
$\rho_r$	beta	0.500	0.1000	0.562	0.0612	0.4588	0.6575
$\rho_q$	beta	0.400	0.2000	0.319	0.1018	0.1580	0.4865
$\rho_{\pi^*}$	beta	0.800	0.1000	0.456	0.0611	0.3559	0.5568
$\rho_{y^*}$	beta	0.900	0.1000	0.749	0.0509	0.6670	0.8321
$\rho_z$	beta	0.200	0.0500	0.241	0.0373	0.1817	0.3025
$\epsilon_r$	invg	0.500	4.0000	0.094	0.0133	0.0725	0.1132
$\epsilon_q$	invg	1.500	4.0000	0.198	0.0090	0.1863	0.2108

(Continued on next page)

Table 3: (continued)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
$\epsilon_{y^*}$	invga	1.500	4.0000	0.777	0.3060	0.3912	1.2052
$\epsilon_{\pi^*}$	invga	0.500	4.0000	0.477	0.0297	0.4283	0.5253
$\epsilon_z$	invga	1.000	4.0000	0.263	0.0537	0.1791	0.3481

Table 4: Results from Metropolis-Hastings for W3 Component

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
$\kappa$	gamma	0.500	0.2500	0.378	0.0778	0.2536	0.5016
$\psi_1$	gamma	1.500	0.5000	2.769	0.4317	2.0326	3.4240
$\psi_2$	gamma	0.250	0.1500	0.179	0.0769	0.0484	0.3062
$\psi_3$	gamma	0.250	0.1500	0.175	0.0473	0.0960	0.2471
$\tau$	beta	0.500	0.2000	0.690	0.0692	0.5749	0.8069
$\rho_r$	beta	0.500	0.1000	0.668	0.0423	0.6000	0.7414
$\rho_q$	beta	0.400	0.2000	0.708	0.1219	0.5238	0.9107
$\rho_{\pi^*}$	beta	0.800	0.1000	0.831	0.0423	0.7592	0.9013
$\rho_{y^*}$	beta	0.900	0.1000	0.914	0.0385	0.8552	0.9816
$\rho_z$	beta	0.200	0.0500	0.437	0.0335	0.3840	0.4927
$\epsilon_r$	invga	0.500	4.0000	0.076	0.0072	0.0641	0.0873
$\epsilon_q$	invga	1.500	4.0000	0.189	0.0025	0.1863	0.1922
$\epsilon_{y^*}$	invga	1.500	4.0000	0.804	0.2895	0.3758	1.2188
$\epsilon_{\pi^*}$	invga	0.500	4.0000	0.227	0.0143	0.2039	0.2504
$\epsilon_z$	invga	1.000	4.0000	0.179	0.0246	0.1402	0.2164

Table 5: Results from Metropolis-Hastings for W4 Component

		Prior		Posterior			
		Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf HPD sup
$\kappa$	gamma	0.500	0.2500	0.285	0.0514	0.2019	0.3716
$\psi_1$	gamma	1.500	0.5000	2.604	0.2640	2.2327	2.9773
$\psi_2$	gamma	0.250	0.1500	0.035	0.0177	0.0072	0.0621
$\psi_3$	gamma	0.250	0.1500	0.054	0.0248	0.0146	0.0934
$\tau$	beta	0.500	0.2000	0.571	0.0635	0.4682	0.6735
$\rho_r$	beta	0.500	0.1000	0.521	0.0431	0.4685	0.5917
$\rho_q$	beta	0.400	0.2000	0.566	0.1345	0.3478	0.7560
$\rho_{\pi^*}$	beta	0.800	0.1000	0.918	0.0271	0.8730	0.9614
$\rho_{y^*}$	beta	0.900	0.1000	0.979	0.0135	0.9608	0.9999
$\rho_z$	beta	0.200	0.0500	0.323	0.0479	0.2577	0.3903
$\epsilon_r$	invga	0.500	4.0000	0.072	0.0048	0.0636	0.0794
$\epsilon_q$	invga	1.500	4.0000	0.189	0.0022	0.1863	0.1915
$\epsilon_{y^*}$	invga	1.500	4.0000	0.498	0.1400	0.2987	0.7116
$\epsilon_{\pi^*}$	invga	0.500	4.0000	0.080	0.0050	0.0723	0.0885
$\epsilon_z$	invga	1.000	4.0000	0.355	0.0808	0.2407	0.4824

**Table 6. Bayes Factors**

	Aggregate	W1	W2	W3	W4
Marginal Data Densities ( $\psi_3 > 0$ )	-389.94	198.3	474.65	597.42	711.97
Marginal Data Densities ( $\psi_3 = 0$ )	-397.72	197.8	466.55	573.23	611.09
Bayes Factor	2381.810	1.708	3291.13	exp(24.18)	exp(100.87)