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Forecasting using a Nonlinear DSGE Model

Sergey Ivashchenko* and Rangan Gupta**

Abstract

A medium-scale nonlinear dynamic stochastic general equilibrium (DSGE) model was estimated (54 variables, 29 state variables, 7 observed variables). The model includes an observed variable for stock market returns. The root-mean square error (RMSE) of the in-sample and out-of-sample forecasts was calculated. The nonlinear DSGE model with measurement errors outperforms AR (1), VAR (1) and the linearised DSGE in terms of the quality of the out-of-sample forecasts. The nonlinear DSGE model without measurement errors is of a quality equal to that of the linearised DSGE model.

Keywords: nonlinear DSGE; Quadratic Kalman Filter; out-of-sample forecasts.

JEL-codes: E32; E37; E44; E47.

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1. Introduction

One of the most popular approaches for analysis of the macroeconomic environment is the use of dynamic stochastic general equilibrium (DSGE) models. This type of model is the basis of modern macroeconomic theory and is widely used by central banks and other policy-making institutions (Tovar, 2009). DSGE models have a strong microeconomic foundation. The advantage of such an approach is a description of models in terms of 'deep structural' parameters that are not influenced by economic policy (Wickens, 2008). The usage of DSGE models requires knowledge about their behaviour, which depends on parameter values. Different econometric techniques are employed for model estimation, but the empirical literature has focused on the estimation of first-order linearised DSGE models (Tovar, 2009).

Computation with linear approximation is much faster than higher-order approximation, but its behaviour can differ from that of more accurate approximations (see Collard and Juillard, 2001). Second-order approximation can make the difference between the behaviour of models and that of approximation much smaller. Nonlinear approximations of DSGE models have several other advantages: in particular, they allow uncertainty to influence economic choices (Ruge-Murcia, 2012). The likelihood function is sharper for nonlinear approximations, which means a more accurate estimation of the parameters (An and Schorfheide, 2007; Fernandez-Villaverde et al., 2010).

Because of these advantages of nonlinear estimation, forecasting is likely to be of high quality. Many studies demonstrate the high quality forecasting of the linear approximations of DSGE models (Adolfson et al., 2007; Smets and Wouters, 2004). A large portion of models forecast a small number of variables (Rubaszek and Skrzypczynski, 2008; Del Negro and Schorfheide, 2012). However, in some studies, the linearised DSGE model outperform VAR and AR models in terms of out-of-sample forecasting with a large number of variables (Ivashchenko, 2013).

In a few studies, small-scale nonlinear DSGE models are estimated (Pichler, 2008; Gust et al., 2012; Fernandez-Villaverde et al., 2010). Most of them use only three observed variables: output, the nominal interest rate and inflation (Amisano and Tristani, 2010; Pichler, 2008; Balcilar et al., 2013; Gust et al., 2012). A few studies use other observed variables (Doh, 2011, uses additional data about the yield curve; Hall, 2012, uses consumption instead of output). The particle filter is used in the studies described above. New results demonstrate the great advantage of alternative approaches over the use of particle filters (Andreasen, 2008; Ivashchenko, 2014; Kollmann, 2014).

Most estimated small-scale nonlinear DSGE models do not provide information about out-of-sample forecasts quality. The forecasting quality of a nonlinear DSGE is nearly the same (or slightly worse) than that of a linearised DSGE model according to Pichler (2008) (this is virtually the only study that discusses the out-of-sample forecasts of estimated DSGE models), but the corresponding model does not include observed variables, which are sensitive to nonlinearities.

This study presents an estimated medium-scale nonlinear DSGE model with seven observed variables, including stock market returns. The DSGE model is described in section 2. Section 3 presents information on estimation techniques and the data utilized. Section 4 describes the estimation results and the quality of forecasts (in-sample and out-of-sample). Section 5 presents some conclusions.

2. Model

The DSGE model includes four types of agents: householders, firms, the government and the foreign sector. The structure of the model is presented in Figure 1. The DSGE model includes central New-Keynesian features (for example, sticky price and adjustment costs in investment).

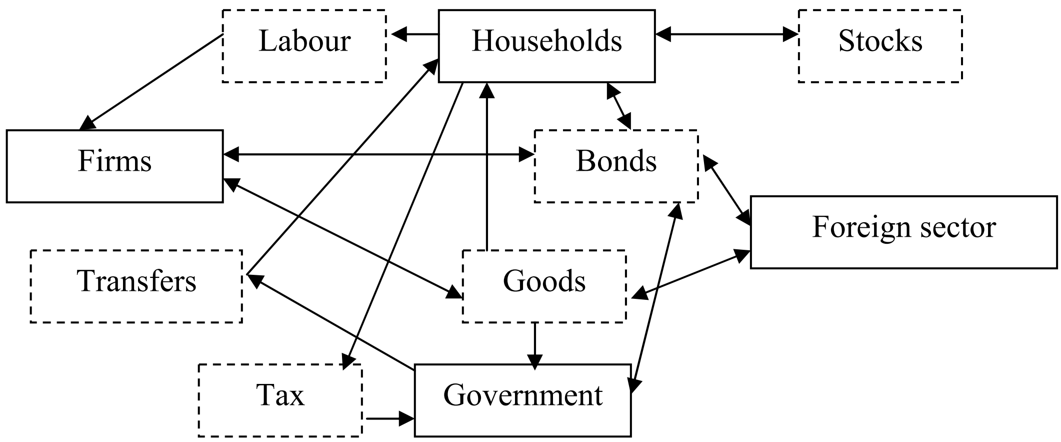

Figure 1. Structure of DSGE model

Table 1. The DSGE model variables

Variable	Description	Stationary variable
$B_{F,t}$	Value of bonds bought by firms in period t	$b_{F,t} = {B_{F,t}}/{P_t Z_t}$
$B_{G,t}$	Value of bonds bought by government in period t	$b_{G,t} = {B_{G,t}}/{P_t Z_t}$
$B_{H,t}$	Value of bonds bought by households in period t	$b_{H,t} = {B_{H,t}}/{P_t Z_t}$
$B_{W,t}$	Value of bonds bought by foreign sector in period t	$b_{W,t} = {B_{W,t}}/{P_t Z_t}$
C_t	Consumption at time t	$c_t = \ln(C_t / Z_t)$
D_t	Dividends at time t	$d_t = (C_t / Z_t)$
G_t	Government expenditure at time t	$g_t = \ln(G_t / Z_t)$
H_t	Habit at time t	$h_t = \ln(H_t / Z_t)$
I_t	Investments at time t	$i_t = \ln(I_t / Z_t)$
K_t	Capital at time t	$k_t = \ln(K_t / Z_t)$
L_t	Labour at time t	$l_t = \ln(L_t)$
M_t	Money stock in period t	$m_t = \ln\left({M_t}/{P_t Z_t}\right)$
NX_t	Net export in period t	$nx_t = (NX_t / Z_t)$
P_t	Price of goods in period t	$p_t = \ln(P_t / P_{t-1})$
$P_{F,t}$	Price for goods of firm F in period t	$p_{F,t} = \ln(P_{F,t} / P_t)$
R_t	Interest rate in period t	$r_t = \ln(R_t)$
S_t	Price of stocks in period t	$s_t = \ln\left({S_t}/{P_t Z_t}\right)$
τ_t	Tax rate in period t	$\tau_t = \tau_t$
$T_{TR,t}$	Transfer from government in period t	$\tau_{TR,t} = \ln\left({T_{TR,t}}/{P_t Z_t}\right)$
W_t	Wage in period t	$w_t = \ln\left({W_t}/{P_t Z_t}\right)$
X_t	Amount of stocks bought by householders in period t	$x_t = X_t$
$Y_{D,t}$	Aggregate demand in period t	$y_{D,t} = \ln(Y_{D,t} / Z_t)$
$Y_{F,t}$	Output of firm F in period t	$y_{F,t} = \ln(Y_{F,t} / Z_t)$
$Z_{\alpha,t}$	Exogenous process corresponding to elasticity of production function	$z_{\alpha,t} = Z_{\alpha,t}$
$Z_{\beta,t}$	Exogenous process corresponding to intertemporal preferences of households	$z_{\beta,t} = \ln(Z_{\beta,t} / Z_{\beta,t-1})$
$Z_{BF,t}$	Exogenous process corresponding to conventional level of debt pressure	$z_{BF,t} = Z_{BF,t}$

$Z_{BH,t}$	Exogenous process corresponding to stickiness of households' bond position	$z_{BH,t} = \ln\left(Z_{BH,t} / Z_t^{1-\omega_C}\right)$
$Z_{G,t}$	Exogenous process corresponding to government expenditure	$z_{G,t} = \ln\left(Z_{G,t}\right)$
$Z_{I,t}$	Exogenous process corresponding to decreasing efficiency of investments	$z_{I,t} = \ln\left(Z_{I,t}\right)$
$Z_{L,t}$	Exogenous process corresponding to households' amount of labour	$z_{L,t} = \ln\left(Z_{L,t} / Z_t^{1-\omega_C}\right)$
$Z_{M,t}$	Exogenous process corresponding to liquidity preferences of households	$z_{M,t} = \ln\left(Z_{M,t} / Z_t^{-\omega_C}\right)$
$Z_{NX,t}$	Exogenous process corresponding to net export	$z_{NX,t} = Z_{NX,t}$
$Z_{P,t}$	Exogenous process corresponding to level of price stickiness	$z_{P,t} = \ln\left(Z_{P,t}\right)$
$Z_{R,t}$	Exogenous process corresponding to monetary policy	$z_{R,t} = Z_{R,t}$
$Z_{\tau,t}$	Exogenous process corresponding to taxation policy	$z_{\tau,t} = Z_{\tau,t}$
$Z_{TR,t}$	Exogenous process corresponding to transfers policy	$z_{TR,t} = Z_{TR,t}$
Z_t	Exogenous process corresponding to technological development	$z_t = \ln\left(Z_t / Z_{t-1}\right)$

2.1 Householders

Households maximize the expected sum of their discounted utility functions (1) with budget restriction (2). Householders do not own capital, but they can invest in domestic stocks and bonds as a means of saving money. The utility function consists of the propensity to consume with a habit effect, the disutility of labour, money at the utility function, and the disutility of bond position deviation from preferred level.

$$E\left(\sum_{t=0}^{\infty} Z_{\beta,t}\left(\begin{array}{l}\dfrac{\left(C_t - h_C H_{t-1}\right)^{1-\omega_C}}{1-\omega_C} - Z_{L,t}\dfrac{L_t^{1+\omega_L}}{1+\omega_L} \\ + Z_{M,t}\dfrac{M_t}{P_t} - Z_{BH,t}\left(\dfrac{B_{H,t}}{P_t Z_t} - \mu_B\right)^2\end{array}\right)\right) \rightarrow \max_{B,C,L,M,X} \tag{1}$$

$$P_t C_t + M_t + B_{H,t} + X_t S_t = (1-\tau_t) W_t L_t + M_{t-1} + R_{t-1} B_{H,t-1} + X_{t-1}\left(S_t + D_t\right) + T_{TR,t} \quad (2),$$

where C_t is consumption in period t, L_t is labour supply in period t, M_t is money stock in period t, P_t is the price of goods in period t, $B_{H,t}$ is the value of bonds bought by householders in period t, S_t is the price of stocks in period t, X_t is the amount of stocks bought by householders in period t, τ_t is the tax rate in period t, $T_{TR,t}$ is the transfer from government in period t, R_t is the interest rate on bonds in period t, and D_t is the dividends of stocks in period t.

2.2 Finished goods-producing firms

Perfectly competitive firms produce the final good Y_t using the intermediate goods $Y_{j,t}$ and the CES production technology:

$$Y_t = \left(\int_0^1 Y_{j,t}^{(\theta-1)/\theta} dj \right)^{\theta/(\theta-1)} \tag{3}$$

Profit maximization and zero profit condition for the finished goods producers imply the following price level P_t and demand function for the intermediate good, j:

$$Y_{j,t} = \left(\frac{P_{j.t}}{P_t} \right)^{-\theta} Y_t \tag{4}$$

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta} dj \right)^{1/(1-\theta)} \tag{5}$$

2.3 Intermediate goods-producing firms

Firms maximize their expected discounted utility function (6) with restrictions. The utility function consists of dividends flow and two rigidities (stickiness of bond position and price stickiness in the Rotemberg form – Lombardo and Vestin, 2008). Firms are working in a market with monopolistic competition; therefore, they have a demand restriction (7). The budget restriction (8) and production function (9) is common. The restriction of capital evolution (10) contains investment rigidity.

$$E\left(\sum_{t=0}^{\infty}\left(\prod_{k=0}^{t-1} R_k\right)^{-1}\left(\begin{array}{l} D_t - P_{F,t}Y_{F,t}\mu_{FB}\left(\dfrac{B_{F,t}}{P_t Z_t} - Z_{BF,t}\right) \\ - P_{F,t}Y_{F,t}Z_{P,t}\left(\dfrac{P_{F,t}}{P_{F,t-1}} - \bar{p}\right)^2 \end{array}\right)\right) \to \max_{D,B,P,Y,K,I,L} \tag{6}$$

$$Y_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{-\theta}\left(Y_{D,t}\right) \tag{7}$$

$$D_t + P_t I_t + W_t L_t + B_{F,t} = P_{F,t}Y_{F,t} + R_{t-1}B_{F,t-1} \tag{8}$$

$$Y_{F,t} = \left(Z_t L_t\right)^{Z_{\alpha,t}}\left(K_{t-1}\right)^{1-Z_{\alpha,t}} \tag{9}$$

$$K_t = (1-\delta)K_{t-1} + I_t\left(1 - Z_{I,t}\left(\frac{I_t}{I_{t-1}} - \bar{y}\right)^2\right) \tag{10}$$

, where D_t is the dividends of the firm in period t, $Y_{F,t}$ is the output of firm F in period t, $P_{F,t}$ is the price of goods for firm F in period t, I_t is the demand for investments goods in period t, $Y_{D,t}$ is the aggregate demand in period t, P_t is the price level for domestic goods in period t, $B_{F,t}$ is the value of bonds bought by the firm in period t, K_t is the amount of capital used by the firm in period t, and L_t is the amount of labour used by the firm in period t.

2.4 Government, foreign sector and balance equations

The government makes its decisions according to policy rules and budgetary restrictions. It has the following budgetary restriction:

$$P_t G_t + T_{TR,t} + B_{G,t} + M_{t-1} = \tau_t W_t L_t + R_{t-1}B_{G,t-1} + M_t \tag{11}$$

The monetary policy rule is as follows:

$$\ln(R_t) = \gamma_R \ln(R_{t-1}) + (1-\gamma_R)\left(\gamma_{RP}\left(\ln\left(\frac{P_t}{P_{t-1}}\right) - \bar{p}\right) + \gamma_{RY}\left(\ln\left(\frac{Y_{D,t}}{Y_{D,t-1}}\right) - \bar{y}\right) + Z_{R,t}\right) \quad (12)$$

The fiscal policy rules are as follows:

$$\ln\left(\frac{G_t}{Y_{D,t}}\right) = \gamma_G \ln\left(\frac{G_{t-1}}{Y_{D,t-1}}\right) + (1-\gamma_G)\left(\begin{array}{l} Z_{G,t} + \gamma_{GB}\left(\frac{B_{G,t}}{P_t Z_t} - \bar{b}_G\right) + \\ + \gamma_{GY}\left(\ln\left(\frac{Y_{D,t}}{Y_{D,t-1}}\right) - \bar{y}\right) \end{array}\right) \quad (13)$$

$$\ln\left(\frac{T_{TR,t}}{Y_{D,t}}\right) = \gamma_{TR} \ln\left(\frac{T_{TR,t-1}}{Y_{D,t-1}}\right) + (1-\gamma_{TR})\left(\begin{array}{l} Z_{TR,t} + \gamma_{TRB}\left(\frac{B_{G,t}}{P_t Z_t} - \bar{b}_G\right) + \\ + \gamma_{TRY}\left(\ln\left(\frac{Y_{D,t}}{Y_{D,t-1}}\right) - \bar{y}\right) \end{array}\right) \quad (14)$$

$$\tau_t = \gamma_T \tau_{t-1} + (1-\gamma_T)\left(\gamma_{TB}\left(\frac{B_{G,t}}{P_t Z_t} - \bar{b}_G\right) + \gamma_{TY}\left(\ln\left(\frac{Y_{D,t}}{Y_{D,t-1}}\right) - \bar{y}\right) + Z_{T,t}\right) \quad (15)$$

The foreign sector is exogenous. It has a budgetary restriction (16) and is subject to an exogenous rule (17).

$$NX_t P_t + B_{W,t} = R_{t-1} B_{W,t-1} \quad (16)$$

$$\left(\frac{NX_t}{Z_t}\right) = \gamma_{NX}\left(\frac{NX_{t-1}}{Z_{t-1}}\right) + (1-\gamma_{NX})\left(\gamma_{NXB}\left(\frac{B_{W,t}}{P_t Z_t} - \bar{b}_W\right) + Z_{NX,t}\right) \quad (17)$$

The three balance restrictions are as follows: each bond should be bought by someone (18), the amount of stocks is equal to one (19), and aggregate demand consists of consumption, investments, government consumption and net exports (20). Formula (21) denotes how the habit is formed.

$$B_{H,t} + B_{F,t} + B_{G,t} + B_{W,t} = 0 \tag{18}$$

$$X_t = 1 \tag{19}$$

$$Y_{D,t} = C_t + I_t + G_t + NX_t \tag{20}$$

$$H_t = h_h H_{t-1} + C_t \tag{21}$$

All the exogenous processes are AR (1) with the following parameterization:

$$z_{*,t} = \eta_{0,*,t}(1 - \eta_{1,*,t}) + \eta_{1,*,t} z_{*,t-1} + \varepsilon_{*,t} \tag{22}$$

3. Estimation

Of the methods used for non-linear approximations of DSGE models, the perturbation method is the most widely used (Schmitt-Grohe and Uribe, 2004), and hence, it is used in this study. The maximum likelihood method is used for parameters estimation.

A few nonlinear filters can be used to calculate the likelihood function. One is the particle filter, which is used in most studies estimating nonlinear DSGE models (Pichler, 2008; Hall, 2012; Doh, 2011). However, it is too slow for implementation with medium-scale models. Another is the central difference Kalman filter (CDKF), which outperforms the particle filter (Andreasen, 2008). However, the quadratic Kalman filter (QKF) was used for the likelihood calculation because it produces a better quality of parameters estimations than the CDKF (Ivashchenko, 2014). The QKF was slightly slower than the CDKF (Ivashchenko, 2014), but after the program code was improved, it became six times faster and outperformed the CDKF in terms of speed.

The QKF is based on a normal approximation of density. Approximation with the perturbation method produces equation (23), which describes the data generating process for state variables (X_t). Equation (24) describes the dependence between observed variables (Y_t) and state variables. Exogenous shocks (ε_t) and measurement errors (u_t) have a normal distribution with zero mean and covariance matrices, Ω_ε and Ω_u.

$$X_t = \begin{bmatrix} B_X & B_\varepsilon \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \varepsilon_t \end{bmatrix} + C + \begin{bmatrix} A_{xx} & A_{x\varepsilon} & 0 & A_{\varepsilon\varepsilon} \end{bmatrix} \begin{bmatrix} X_{t-1} \otimes X_{t-1} \\ X_{t-1} \otimes \varepsilon_t \\ \varepsilon_t \otimes X_{t-1} \\ \varepsilon_t \otimes \varepsilon_t \end{bmatrix} \quad (23)$$

$$Y_t = S + DX_t + u_t \quad (24)$$

The updating step is similar with the Kalman filter, owing to the linearity of equation (24). The prediction step is based on an assumption of normal distribution of the state variables vector (X_{t-1}). The expected value of vector X_t is a function of the mean and covariance of vectors X_{t-1} and ε_t. The covariance of vector X_t is a function of the first, second, third, and fourth moments of vectors X_{t-1} and ε_t. However, the third and fourth moments of a vector with a normal distribution are a function of the mean and covariance. Thus, the QKF computes the first and second moments of the state variables vector and assumes that it has a normal distribution.

An alternative approach for nonlinear approximation is the pruning method (Kim et al., 2008), for which there is a nonlinear filter (Kollmann, 2014). It is faster than the QKF (before optimization of the program code) for small-scale models, but is much slower (by about five times) for medium-scale (with 20 state variables) models (Kollmann, 2014). The DSGE model described above has 54 variables (29 state variables); this was an additional reason for the usage of the QKF.

The model was estimated with quarterly data from the USA since 1985Q1 until 2013Q2. The following observed variables are used: logarithm of consumption as a fraction of GDP (obs_C); logarithm of government expenditure as a fraction of GDP (obs_G); logarithm of compensation of employees as a fraction of GDP (obs_{WL}); three-month euro-dollar deposit rate (obs_R); GDP growth rate (obs_Y); growth rate of the GDP deflator (obs_P); and MSCI USA gross return (obs_{STR}). The DSGE model was estimated four times (linearised model with the Kalman filter and second-order approximation with the QKF; with and without measurement errors for obs_{STR}).

4. Results

The estimation results are presented in Table 2. Some interesting details regarding these are as follows. The monetary policy parameter γ_{RP} is less than 1. Many studies have valued this parameter at greater than 1 (1.045 – Fernandez-Villaverde et al., 2010; 1.66 – Smets and Wouters, 2004; 5.0 – Gust et al., 2012), but in others, it is less than 1 (0.63 – nonlinear estimation, Hall, 2012). Low values of γ_{RP} require additional comments: the log-likelihood value of the DSGE model with restriction (γ_{RP}>1) is less than 2900, which is much worse than with the other estimations (the QKF without measurement errors is 2947.5; the QKF with measurement errors is 2986.1; the line estimation without measurement errors is 2920.8; and the line estimation with measurement errors is 2986.9). The OLS estimation of the monetary policy rule (from 1990Q1) produces γ_{RP}=0.39.

Table 2. The DSGE model estimation results

	QKF				line			
	without measur.er.		with measur. error		without measur.er.		with measur. error	
Param.	value	std	value	std	value	std	value	std
std ε_{α}	2.60x10^{-01}	2.16x10^{-02}	2.95x10^{-01}	3.27x10^{-02}	8.02x10^{-02}	1.84x10^{-02}	6.83x10^{-02}	1.39x10^{-02}
std ε_{β}	5.91x10^{-10}	2.26x10^{-05}	3.77x10^{-08}	2.12x10^{-05}	7.10x10^{-07}	3.55x10^{-05}	3.11x10^{-05}	2.09x10^{-05}
std ε_{BF}	7.15x10^{-02}	1.01x10^{-01}	1.63x10^{-01}	5.74x10^{-02}	4.57x10^{-01}	2.45x10^{-01}	2.50x10^{+00}	3.04x10^{+00}
std ε_{BH}	5.52x10^{-01}	1.16x10^{-01}	4.65x10^{-01}	1.60x10^{-01}	2.86x10^{-04}	3.87x10^{+07}	2.46x10^{+00}	1.71x10^{+00}
std ε_{G}	2.32x10^{-02}	3.42x10^{-03}	2.65x10^{-02}	6.84x10^{-03}	3.55x10^{-02}	1.08x10^{-02}	1.36x10^{-07}	2.09x10^{-05}
std ε_{I}	3.55x10^{-08}	2.26x10^{-05}	1.92x10^{-09}	2.12x10^{-05}	4.84x10^{-04}	3.87x10^{+07}	4.61x10^{-04}	2.74x10^{+03}
std ε_{L}	2.25x10^{-08}	2.26x10^{-05}	1.13x10^{-08}	2.12x10^{-05}	1.00x10^{+02}	4.40x10^{-05}	7.85x10^{+01}	1.20x10^{+01}
std ε_{M}	1.77x10^{+00}	5.06x10^{-01}	8.70x10^{-01}	1.57x10^{-01}	1.00x10^{+02}	4.40x10^{-05}	5.60x10^{-01}	4.58x10^{-01}
std ε_{NX}	3.70x10^{+00}	9.97x10^{-01}	4.07x10^{+00}	1.08x10^{+00}	2.55x10^{-01}	8.46x10^{-02}	8.41x10^{-01}	7.94x10^{-02}
std ε_{P}	1.04x10^{-01}	4.24x10^{-02}	8.54x10^{-02}	1.75x10^{-02}	6.22x10^{-04}	5.60x10^{+01}	4.61x10^{-04}	1.59x10^{+03}
std ε_{R}	1.93x10^{-02}	3.34x10^{-03}	2.02x10^{-02}	2.22x10^{-03}	2.69x10^{-02}	5.06x10^{-03}	2.14x10^{-02}	3.90x10^{-03}
std ε_{τ}	2.76x10^{-02}	7.52x10^{-03}	2.05x10^{-08}	2.12x10^{-05}	5.96x10^{-02}	9.97x10^{-03}	3.83x10^{-02}	7.79x10^{-03}
std ε_{TR}	2.28x10^{-08}	2.26x10^{-05}	4.75x10^{-09}	2.12x10^{-05}	2.66x10^{-06}	3.55x10^{-05}	5.33x10^{-06}	2.09x10^{-05}
std ε_{Y}	3.42x10^{-03}	2.65x10^{-04}	3.38x10^{-03}	1.82x10^{-04}	3.51x10^{-03}	4.19x10^{-04}	2.45x10^{-03}	3.07x10^{-04}
std obs_{STR}	-	-	6.70x10^{-02}	4.17x10^{-03}	-	-	6.60x10^{-02}	5.01x10^{-03}
γ_{NX}	2.09x10^{-01}	2.44x10^{-01}	3.05x10^{-01}	2.70x10^{-01}	-2.21x10^{-01}	6.76x10^{-02}	-2.48x10^{-01}	1.22x10^{-01}
γ_{NXB}	5.00x10^{+00}	7.56x10^{-05}	3.68x10^{+00}	1.18x10^{+00}	3.03x10^{-01}	5.32x10^{-02}	2.25x10^{-01}	1.16x10^{-01}
γ_{G}	7.95x10^{-01}	2.18x10^{-02}	8.26x10^{-01}	1.77x10^{-02}	8.68x10^{-01}	2.02x10^{-02}	8.47x10^{-01}	2.86x10^{-02}
γ_{GB}	1.62x10^{+00}	2.05x10^{-01}	1.62x10^{+00}	3.58x10^{-01}	4.24x10^{-01}	6.32x10^{-02}	3.53x10^{-01}	2.34x10^{-01}
γ_{GY}	-2.41x10^{+00}	3.97x10^{-01}	-4.49x10^{+00}	3.70x10^{-05}	-5.00x10^{+00}	3.29x10^{-05}	-5.00x10^{+00}	2.33x10^{-05}
γ_{TR}	7.81x10^{-01}	6.61x10^{-02}	8.44x10^{-01}	4.10x10^{-02}	9.78x10^{-01}	9.71x10^{-03}	9.92x10^{-01}	9.80x10^{-03}
γ_{TRB}	-3.65x10^{-02}	1.20x10^{-01}	-4.28x10^{-01}	3.12x10^{-01}	6.51x10^{-01}	2.07x10^{-01}	-1.35x10^{-01}	2.96x10^{-01}
γ_{TRY}	-4.63x10^{+00}	9.94x10^{-01}	2.47x10^{+00}	9.30x10^{-01}	-5.00x10^{+00}	3.29x10^{-05}	5.00x10^{+00}	1.75x10^{-05}
γ_{τ}	8.49x10^{-01}	1.26x10^{-02}	8.72x10^{-01}	1.57x10^{-02}	8.78x10^{-01}	1.88x10^{-02}	8.67x10^{-01}	1.88x10^{-02}
$\gamma_{\tau B}$	3.73x10^{-01}	8.31x10^{-02}	2.40x10^{-01}	8.37x10^{-02}	2.00x10^{-01}	4.45x10^{-02}	7.29x10^{-02}	4.57x10^{-02}
$\gamma_{\tau Y}$	-2.60x10^{+00}	2.74x10^{-01}	-2.88x10^{+00}	3.65x10^{-01}	-3.29x10^{+00}	2.34x10^{-01}	-4.54x10^{+00}	5.16x10^{-01}

γ_{R}	$8.80x10^{-01}$	$2.11x10^{-02}$	$9.05x10^{-01}$	$1.52x10^{-02}$	$9.29x10^{-01}$	$1.64x10^{-02}$	$9.06x10^{-01}$	$2.04x10^{-02}$
γ_{RP}	$1.15x10^{-01}$	$5.64x10^{-02}$	$2.91x10^{-02}$	$1.55x10^{-01}$	$3.90x10^{-03}$	$3.07x10^{-02}$	$6.84x10^{-10}$	$2.09x10^{-05}$
γ_{RY}	$7.54x10^{-01}$	$1.24x10^{-01}$	$1.17x10^{+00}$	$3.08x10^{-01}$	$9.06x10^{-01}$	$2.12x10^{-01}$	$8.08x10^{-01}$	$1.50x10^{-01}$
h_{C}	$6.33x10^{-01}$	$1.80x10^{-05}$	$6.34x10^{-01}$	$3.59x10^{-05}$	$6.29x10^{-01}$	$2.80x10^{-05}$	$6.31x10^{-01}$	$2.23x10^{-05}$
h_{h}	$3.50x10^{-01}$	$1.68x10^{-04}$	$3.50x10^{-01}$	$1.24x10^{-04}$	$3.52x10^{-01}$	$2.78x10^{-05}$	$3.45x10^{-01}$	$2.19x10^{-05}$
μ_{F}	$-1.13x10^{-03}$	$8.15x10^{-03}$	$-2.17x10^{-03}$	$7.66x10^{-01}$	$-5.47x10^{-08}$	$5.11x10^{+00}$	$6.36x10^{-08}$	$6.35x10^{+01}$
μ_{H}	$5.79x10^{+00}$	$1.01x10^{-01}$	$6.01x10^{+00}$	$4.32x10^{-01}$	$5.28x10^{+00}$	$2.53x10^{-03}$	$1.08x10^{+00}$	$8.89x10^{-01}$
$\eta_{0,\alpha}$	$5.99x10^{-01}$	$4.00x10^{-05}$	$5.99x10^{-01}$	$5.30x10^{-05}$	$6.00x10^{-01}$	$4.83x10^{-05}$	$5.97x10^{-01}$	$2.23x10^{-05}$
$\eta_{0,\beta}$	$-2.00x10^{-02}$	$2.44x10^{-05}$	$-2.00x10^{-02}$	$2.15x10^{-05}$	$-9.33x10^{-03}$	$2.93x10^{-05}$	$-1.98x10^{-02}$	$2.38x10^{-05}$
$\eta_{0,BF}$	$-8.80x10^{+00}$	$7.85x10^{-02}$	$-8.74x10^{+00}$	$2.96x10^{-01}$	$-1.26x10^{+01}$	$4.86x10^{-03}$	$-3.78x10^{+00}$	$8.70x10^{-01}$
$\eta_{0,BH}$	$1.94x10^{+01}$	$2.12x10^{+00}$	$1.92x10^{+01}$	$2.04x10^{+01}$	$1.95x10^{+01}$	$4.93x10^{-02}$	$1.30x10^{+00}$	$1.03x10^{+00}$
$\eta_{0,G}$	$-2.00x10^{+00}$	$5.07x10^{-05}$	$-2.00x10^{+00}$	$2.55x10^{-05}$	$-1.98x10^{+00}$	$1.02x10^{-03}$	$-1.95x10^{+00}$	$1.06x10^{-02}$
$\eta_{0,I}$	$1.84x10^{+01}$	$2.22x10^{+00}$	$1.19x10^{+01}$	$2.31x10^{+00}$	$1.21x10^{+01}$	$2.62x10^{+00}$	$7.79x10^{+00}$	$2.46x10^{+00}$
$\eta_{0,L}$	$2.78x10^{+00}$	$1.10x10^{-01}$	$2.84x10^{+00}$	$9.11x10^{-01}$	$3.94x10^{+00}$	$2.81x10^{-05}$	$4.14x10^{+00}$	$2.31x10^{-05}$
$\eta_{0,M}$	$-1.84x10^{+00}$	$2.15x10^{-01}$	$-1.71x10^{+00}$	$9.92x10^{-01}$	$-2.75x10^{+00}$	$2.91x10^{-05}$	$-1.43x10^{+00}$	$2.28x10^{-05}$
$\eta_{0,NX}$	$1.18x10^{-01}$	$3.05x10^{-03}$	$1.10x10^{-01}$	$1.57x10^{-02}$	$1.78x10^{-01}$	$2.52x10^{-05}$	$7.42x10^{-02}$	$2.38x10^{-05}$
$\eta_{0,P}$	$6.13x10^{+00}$	$8.96x10^{-02}$	$6.41x10^{+00}$	$9.05x10^{-02}$	$5.33x10^{+00}$	$1.25x10^{-01}$	$4.62x10^{+00}$	$2.11x10^{-01}$
$\eta_{0,R}$	$8.83x10^{-03}$	$2.33x10^{-03}$	$1.14x10^{-02}$	$1.25x10^{-03}$	$1.35x10^{-02}$	$2.71x10^{-05}$	$8.34x10^{-03}$	$2.38x10^{-05}$
$\eta_{0,\tau}$	$6.00x10^{-01}$	$4.12x10^{-05}$	$5.99x10^{-01}$	$5.39x10^{-05}$	$5.49x10^{-01}$	$2.80x10^{-05}$	$4.84x10^{-01}$	$2.21x10^{-05}$
$\eta_{0,TR}$	$-1.77x10^{+00}$	$2.14x10^{-04}$	$-1.77x10^{+00}$	$4.02x10^{-05}$	$-1.98x10^{+00}$	$1.03x10^{-03}$	$-2.27x10^{+00}$	$1.51x10^{-02}$
$\eta_{0,Y}$	$6.24x10^{-03}$	$5.76x10^{-05}$	$6.79x10^{-03}$	$1.37x10^{-04}$	$7.03x10^{-03}$	$2.72x10^{-05}$	$2.65x10^{-03}$	$2.38x10^{-05}$
$\eta_{1,\alpha}$	$9.75x10^{-01}$	$3.04x10^{-03}$	$9.74x10^{-01}$	$2.76x10^{-03}$	$9.47x10^{-01}$	$6.77x10^{-03}$	$9.38x10^{-01}$	$8.19x10^{-03}$
$\eta_{1,\beta}$	$2.55x10^{-02}$	$4.50x10^{-02}$	$6.06x10^{-02}$	$7.31x10^{-02}$	$-1.95x10^{-01}$	$1.28x10^{-01}$	$-2.33x10^{-03}$	$3.00x10^{+00}$
$\eta_{1,BF}$	$9.06x10^{-01}$	$9.72x10^{-02}$	$9.67x10^{-01}$	$3.73x10^{-02}$	$8.05x10^{-01}$	$1.25x10^{-01}$	$9.72x10^{-01}$	$2.84x10^{-02}$
$\eta_{1,BH}$	$4.25x10^{-01}$	$1.56x10^{-01}$	$-2.22x10^{-01}$	$2.25x10^{-01}$	$-4.09x10^{-02}$	$3.73x10^{-02}$	$-1.13x10^{-01}$	$1.26x10^{-01}$
$\eta_{1,G}$	$-2.03x10^{-01}$	$5.43x10^{-02}$	$-2.65x10^{-01}$	$1.70x10^{-01}$	$-1.60x10^{-01}$	$1.30x10^{-01}$	$-1.61x10^{-02}$	$2.88x10^{+00}$
$\eta_{1,I}$	$-2.88x10^{-01}$	$2.21x10^{-01}$	$-1.84x10^{-01}$	$6.03x10^{-02}$	$1.98x10^{-03}$	$3.25x10^{-02}$	$5.54x10^{-03}$	$2.11x10^{+00}$
$\eta_{1,L}$	$1.08x10^{-01}$	$9.81x10^{-02}$	$-5.12x10^{-02}$	$2.03x10^{-01}$	$9.98x10^{-01}$	$4.72x10^{-05}$	$9.98x10^{-01}$	$1.82x10^{-05}$
$\eta_{1,M}$	$9.81x10^{-01}$	$1.19x10^{-02}$	$9.41x10^{-01}$	$1.18x10^{-02}$	$9.88x10^{-01}$	$3.20x10^{-03}$	$-2.01x10^{-01}$	$5.52x10^{-01}$
$\eta_{1,NX}$	$9.86x10^{-01}$	$3.42x10^{-03}$	$9.90x10^{-01}$	$2.17x10^{-03}$	$9.24x10^{-01}$	$2.24x10^{-02}$	$9.93x10^{-01}$	$9.42x10^{-04}$
$\eta_{1,P}$	$-9.32x10^{-03}$	$6.81x10^{-03}$	$-1.21x10^{-02}$	$7.11x10^{-03}$	$-4.51x10^{-05}$	$2.53x10^{-02}$	$5.55x10^{-04}$	$2.94x10^{+00}$
$\eta_{1,R}$	$5.10x10^{-01}$	$7.57x10^{-02}$	$4.06x10^{-01}$	$7.98x10^{-02}$	$4.22x10^{-01}$	$8.86x10^{-02}$	$4.50x10^{-01}$	$8.65x10^{-02}$
$\eta_{1,\tau}$	$5.61x10^{-01}$	$5.98x10^{-02}$	$5.64x10^{-01}$	$2.08x10^{-01}$	$6.29x10^{-02}$	$6.47x10^{-02}$	$1.42x10^{-01}$	$1.31x10^{-01}$
$\eta_{1,TR}$	$9.83x10^{-01}$	$9.17x10^{-03}$	$9.93x10^{-01}$	$5.18x10^{-03}$	$-1.22x10^{-02}$	$4.53x10^{-02}$	$-6.07x10^{-02}$	$2.44x10^{+00}$
$\eta_{1,Y}$	$2.17x10^{-01}$	$2.97x10^{-02}$	$1.66x10^{-01}$	$2.50x10^{-02}$	$1.86x10^{-01}$	$8.40x10^{-02}$	$3.08x10^{-02}$	$1.03x10^{-01}$
ω_{C}	$1.17x10^{+00}$	$4.96x10^{-02}$	$1.15x10^{+00}$	$2.16x10^{-01}$	$1.21x10^{+00}$	$2.88x10^{-05}$	$1.20x10^{+00}$	$2.27x10^{-05}$
ω_{L}	$1.58x10^{-01}$	$5.57x10^{-02}$	$1.85x10^{-02}$	$8.34x10^{-02}$	$2.85x10^{-03}$	$2.58x10^{-05}$	$3.28x10^{-07}$	$2.09x10^{-05}$
δ	$1.00x10^{-02}$	$2.28x10^{-05}$	$1.00x10^{-02}$	$2.11x10^{-05}$	$1.00x10^{-02}$	$2.98x10^{-05}$	$2.70x10^{-02}$	$2.38x10^{-05}$
θ	$7.66x10^{+00}$	$7.65x10^{-04}$	$7.56x10^{+00}$	$5.04x10^{-02}$	$6.35x10^{+00}$	$2.49x10^{-05}$	$1.13x10^{+01}$	$2.32x10^{-05}$

Another important detail of the estimation results is the high values of the standard deviation of the measurement errors (6.6% - QKF, 6.7% - line estimation, 7.2% - standard deviation of obs_{STR}). This could be a result of

MSCI USA properties: it includes international companies (such as APPLE and JOHNSON & JOHNSON), which have a large portion of their production and sales in foreign countries. The identification of a few standard deviations (ε_{BH} and ε_I) is weak with linear approximation. However, this problem does not exist for the QKF. The standard deviation of ε_L is very sensitive to estimation technique (it is high for the linear estimation and almost zero for the QKF). Some autocorrelation coefficients ($\eta_{1,L}$, $\eta_{1,M}$ and $\eta_{1,TR}$) are sensitive to estimation technique as well (they are close to 1 with one estimation technique and close to 0 with another).

Table 3. RMSE of in-sample forecasts

	VAR(1)	AR(1)	DSGE QKF no meas.er.	DSGE QKF meas.er.	DSGE line no meas.er.	DSGE line meas.er.
obsC(+1)	**4.60x10^{-03}**	4.70x10^{-03}	5.29x10^{-03}	4.91x10^{-03}	5.09x10^{-03}	4.66x10^{-03}
obsG(+1)	7.90x10^{-03}	9.33x10^{-03}	**7.88x10^{-03}**	8.03x10^{-03}	9.21x10^{-03}	8.68x10^{-03}
obsY(+1)	**5.00x10^{-03}**	5.49x10^{-03}	5.68x10^{-03}	5.34x10^{-03}	5.72x10^{-03}	5.62x10^{-03}
obsP(+1)	**1.66x10^{-03}**	1.84x10^{-03}	2.22x10^{-03}	1.74x10^{-03}	2.22x10^{-03}	1.93x10^{-03}
obsWL(+1)	**6.49x10^{-03}**	7.17x10^{-03}	9.21x10^{-03}	6.98x10^{-03}	7.54x10^{-03}	6.72x10^{-03}
obsR(+1)	**1.07x10^{-03}**	1.29x10^{-03}	1.15x10^{-03}	1.15x10^{-03}	1.16x10^{-03}	1.13x10^{-03}
obsSTR(+1)	**6.59x10^{-02}**	7.04x10^{-02}	7.51x10^{-02}	7.44x10^{-02}	7.19x10^{-02}	7.22x10^{-02}
obsC(+2)	**5.36x10^{-03}**	5.41x10^{-03}	6.79x10^{-03}	6.57x10^{-03}	6.82x10^{-03}	5.84x10^{-03}
obsG(+2)	**1.26x10^{-02}**	1.58x10^{-02}	1.29x10^{-02}	1.29x10^{-02}	1.63x10^{-02}	1.45x10^{-02}
obsY(+2)	**5.22x10^{-03}**	5.76x10^{-03}	6.22x10^{-03}	5.69x10^{-03}	6.04x10^{-03}	6.13x10^{-03}
obsP(+2)	**1.88x10^{-03}**	2.01x10^{-03}	2.38x10^{-03}	1.96x10^{-03}	2.85x10^{-03}	2.03x10^{-03}
obsWL(+2)	**7.11x10^{-03}**	8.25x10^{-03}	1.16x10^{-02}	8.14x10^{-03}	9.93x10^{-03}	7.89x10^{-03}
obsR(+2)	**1.78x10^{-03}**	2.16x10^{-03}	1.90x10^{-03}	1.90x10^{-03}	1.97x10^{-03}	1.86x10^{-03}
obsSTR(+2)	**6.72x10^{-02}**	7.23x10^{-02}	7.32x10^{-02}	7.28x10^{-02}	7.23x10^{-02}	7.24x10^{-02}
obsC(+3)	**5.87x10^{-03}**	6.01x10^{-03}	8.78x10^{-03}	8.42x10^{-03}	8.51x10^{-03}	7.01x10^{-03}
obsG(+3)	**1.64x10^{-02}**	2.14x10^{-02}	1.72x10^{-02}	1.71x10^{-02}	2.28x10^{-02}	1.97x10^{-02}
obsY(+3)	**5.26x10^{-03}**	6.00x10^{-03}	6.67x10^{-03}	6.09x10^{-03}	6.35x10^{-03}	6.62x10^{-03}
obsP(+3)	**2.01x10^{-03}**	2.15x10^{-03}	2.68x10^{-03}	2.20x10^{-03}	3.57x10^{-03}	2.09x10^{-03}
obsWL(+3)	**7.48x10^{-03}**	9.72x10^{-03}	1.27x10^{-02}	9.31x10^{-03}	1.27x10^{-02}	9.36x10^{-03}
obsR(+3)	**2.40x10^{-03}**	2.90x10^{-03}	2.58x10^{-03}	2.60x10^{-03}	2.70x10^{-03}	2.50x10^{-03}
obsSTR(+3)	**6.76x10^{-02}**	7.26x10^{-02}	7.40x10^{-02}	7.31x10^{-02}	7.26x10^{-02}	7.27x10^{-02}
obsC(+4)	**6.64x10^{-03}**	6.68x10^{-03}	1.08x10^{-02}	1.03x10^{-02}	1.04x10^{-02}	8.39x10^{-03}
obsG(+4)	**1.99x10^{-02}**	2.68x10^{-02}	2.17x10^{-02}	2.17x10^{-02}	2.99x10^{-02}	2.53x10^{-02}
obsY(+4)	**5.32x10^{-03}**	6.05x10^{-03}	6.88x10^{-03}	6.09x10^{-03}	6.59x10^{-03}	6.86x10^{-03}
obsP(+4)	**2.11x10^{-03}**	2.26x10^{-03}	2.99x10^{-03}	2.46x10^{-03}	4.42x10^{-03}	2.27x10^{-03}
obsWL(+4)	**7.72x10^{-03}**	1.11x10^{-02}	1.41x10^{-02}	1.03x10^{-02}	1.62x10^{-02}	1.08x10^{-02}

obsR(+4)	**2.89x10^{-03}**	3.54x10^{-03}	3.15x10^{-03}	3.19x10^{-03}	3.31x10^{-03}	3.00x10^{-03}
obsSTR(+4)	**6.87x10^{-02}**	7.27x10^{-02}	7.43x10^{-02}	7.30x10^{-02}	7.27x10^{-02}	7.29x10^{-02}
average RMSE	**1.48x10^{-02}**	1.65x10^{-02}	1.71x10^{-02}	1.64x10^{-02}	1.76x10^{-02}	1.65x10^{-02}
root mean square RMSE	**2.64x10^{-02}**	2.86x10^{-02}	2.93x10^{-02}	2.88x10^{-02}	2.91x10^{-02}	2.87x10^{-02}
forecasts not worse than VAR	28	0	1	0	0	0
forecasts not worse than AR	28	28	8	16	7	14

The RMSE of the forecasts are presented in Table 3 (in-sample) and Table 4 (out-of-sample). Out-of-sample forecasts were computed for the last 22 quarters (this meant the re-estimation of parameters with dataset without the last quarter (from 1985Q1 until 2013Q1) and the computation of forecasts; the re-estimation without 2 quarters (from 1985Q1 until 2012Q4), and so on; the last re-estimation used dataset without 22 quarters – from 1985Q1 until 2007Q4).

Table 4. RMSE of out-of-sample forecasts

	VAR(1)	AR(1)	DSGE QKF no meas.er.	DSGE QKF meas.er.	DSGE line no meas.er.	DSGE line meas.er.
obsC(+1)	5.23x10^{-03}	**4.72x10^{-03}**	6.48x10^{-03}	5.03x10^{-03}	5.93x10^{-03}	5.76x10^{-03}
obsG(+1)	1.07x10^{-02}	1.29x10^{-02}	**8.64x10^{-03}**	9.20x10^{-03}	9.97x10^{-03}	1.05x10^{-02}
obsY(+1)	7.33x10^{-03}	7.97x10^{-03}	8.05x10^{-03}	**7.27x10^{-03}**	7.91x10^{-03}	8.49x10^{-03}
obsP(+1)	2.19x10^{-03}	2.21x10^{-03}	2.05x10^{-03}	**1.86x10^{-03}**	2.42x10^{-03}	2.24x10^{-03}
obsWL(+1)	1.14x10^{-02}	1.13x10^{-02}	**9.40x10^{-03}**	1.02x10^{-02}	1.12x10^{-02}	1.09x10^{-02}
obsR(+1)	**1.22x10^{-03}**	1.52x10^{-03}	1.64x10^{-03}	1.55x10^{-03}	1.56x10^{-03}	1.55x10^{-03}
obsSTR(+1)	1.01x10^{-01}	**9.79x10^{-02}**	1.07x10^{-01}	1.02x10^{-01}	9.83x10^{-02}	1.01x10^{-01}
obsC(+2)	6.91x10^{-03}	**6.52x10^{-03}**	9.07x10^{-03}	7.40x10^{-03}	9.38x10^{-03}	7.78x10^{-03}
obsG(+2)	1.88x10^{-02}	2.39x10^{-02}	**1.59x10^{-02}**	1.61x10^{-02}	1.93x10^{-02}	1.95x10^{-02}
obsY(+2)	8.31x10^{-03}	8.86x10^{-03}	9.09x10^{-03}	**7.93x10^{-03}**	8.38x10^{-03}	9.65x10^{-03}
obsP(+2)	2.74x10^{-03}	2.51x10^{-03}	**2.24x10^{-03}**	2.27x10^{-03}	2.78x10^{-03}	2.42x10^{-03}
obsWL(+2)	1.04x10^{-02}	1.18x10^{-02}	1.13x10^{-02}	**9.23x10^{-03}**	1.17x10^{-02}	1.24x10^{-02}
obsR(+2)	**1.72x10^{-03}**	2.14x10^{-03}	2.42x10^{-03}	2.34x10^{-03}	2.36x10^{-03}	2.33x10^{-03}
obsSTR(+2)	1.07x10^{-01}	1.02x10^{-01}	1.03x10^{-01}	9.96x10^{-02}	**9.76x10^{-02}**	9.96x10^{-02}
obsC(+3)	8.52x10^{-03}	**7.81x10^{-03}**	1.31x10^{-02}	1.00x10^{-02}	1.29x10^{-02}	1.01x10^{-02}
obsG(+3)	2.78x10^{-02}	3.42x10^{-02}	**2.30x10^{-02}**	2.34x10^{-02}	2.93x10^{-02}	2.93x10^{-02}
obsY(+3)	9.04x10^{-03}	9.47x10^{-03}	9.98x10^{-03}	**8.58x10^{-03}**	9.31x10^{-03}	1.08x10^{-02}
obsP(+3)	3.40x10^{-03}	2.87x10^{-03}	2.83x10^{-03}	**2.80x10^{-03}**	3.62x10^{-03}	2.95x10^{-03}
obsWL(+3)	1.01x10^{-02}	1.37x10^{-02}	1.45x10^{-02}	**1.00x10^{-02}**	1.29x10^{-02}	1.57x10^{-02}
obsR(+3)	**2.00x10^{-03}**	2.56x10^{-03}	2.88x10^{-03}	2.90x10^{-03}	2.92x10^{-03}	2.87x10^{-03}

obsSTR(+3)	1.08×10^{-01}	1.03×10^{-01}	1.08×10^{-01}	1.02×10^{-01}	$\mathbf{1.00 \times 10^{-01}}$	1.02×10^{-01}
obsC(+4)	8.96×10^{-03}	$\mathbf{7.60 \times 10^{-03}}$	1.59×10^{-02}	1.26×10^{-02}	1.62×10^{-02}	1.27×10^{-02}
obsG(+4)	3.78×10^{-02}	4.34×10^{-02}	$\mathbf{2.86 \times 10^{-02}}$	2.96×10^{-02}	3.80×10^{-02}	3.84×10^{-02}
obsY(+4)	9.04×10^{-03}	9.32×10^{-03}	1.04×10^{-02}	$\mathbf{8.58 \times 10^{-03}}$	9.73×10^{-03}	1.13×10^{-02}
obsP(+4)	3.77×10^{-03}	$\mathbf{3.02 \times 10^{-03}}$	3.15×10^{-03}	3.45×10^{-03}	4.20×10^{-03}	3.38×10^{-03}
obsWL(+4)	$\mathbf{1.12 \times 10^{-02}}$	1.78×10^{-02}	2.03×10^{-02}	1.25×10^{-02}	1.75×10^{-02}	1.91×10^{-02}
obsR(+4)	$\mathbf{2.11 \times 10^{-03}}$	3.07×10^{-03}	3.22×10^{-03}	3.38×10^{-03}	3.45×10^{-03}	3.35×10^{-03}
obsSTR(+4)	1.07×10^{-01}	1.02×10^{-01}	1.09×10^{-01}	1.01×10^{-01}	$\mathbf{1.01 \times 10^{-01}}$	1.01×10^{-01}
average RMSE	2.30×10^{-02}	2.34×10^{-02}	2.36×10^{-02}	$\mathbf{2.19 \times 10^{-02}}$	2.32×10^{-02}	2.35×10^{-02}
root mean square RMSE	4.16×10^{-02}	4.06×10^{-02}	4.17×10^{-02}	$\mathbf{3.96 \times 10^{-02}}$	3.96×10^{-02}	4.02×10^{-02}
forecasts not worse than VAR	28	12	11	19	6	9
forecasts not worse than AR	16	28	9	18	14	9

The VAR model produces the best in-sample forecasts; this may be explained by the larger number of parameters (VAR – 84 parameters, AR – 21 parameters, DSGE – 64 or 65 parameters, depending on the existence of measurement errors). The RMSEs of the out-of-sample forecasts are drastically higher than those of the in-sample forecasts because of the financial crisis of 2008-2009. The quality of the in-sample forecast with measurement errors is better for line and quadratic estimations. However, the situation with out-of-sample forecasts is different: line forecasts with measurement errors are worse than without measurement errors.

The in-sample quality of line and quadratic forecasts with measurement errors is nearly the same. Quadratic forecasts with measurement errors outperform all other models in terms of out-of-sample RMSE. It outperforms each of the other models for more than two-thirds of the variables. A comparison of line and quadratic estimation without measurement errors shows a small advantage for line estimation (14 variables forecasts are better than with quadratic estimation of the same model), which is in line with the results of Pichler (2008). It should be noted that forecasts (for 2, 3 and 4 quarters) of stock market returns by the DSGE model outperform the AR and VAR models, despite problems related to international companies.

5. Conclusion

In this study, the medium-scale nonlinear DSGE model was estimated. The DSGE model includes stock market returns, but observed data (MSCI USA gross return) describes international companies. Thus, measurement errors (for the stock returns variable) increase the quality of the model with nonlinear estimation (however, it does not change the quality of the linear estimated model). Measurement errors have a high standard deviation.

The quality of the out-of-sample forecasts of the DSGE models without measurement errors is almost equal (slightly worse) to those of AR (1) and VAR (1) models. The quality of the DSGE model with linear and nonlinear estimations is actually equal. In the case of the existence of measurement errors, the situation is different: the nonlinear DSGE model outperforms all other models (including linearised DSGE). Thus, this study finds that nonlinear DSGE models are more sensitive to misspecification (a negative effect of sharper likelihood), and that achieving an advantage from nonlinear approximation requires a more realistic model than in the case of a linearised model.

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