University of Pretoria
Department of Economics Working Paper Series

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Working Paper: 2016-10
February 2016

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The Efficiency of the Art Market: Evidence from Variance Ratio Tests, Linear and Nonlinear Fractional Integration Approaches

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Abstract

This paper investigates the weak-form efficiency hypothesis for the art market. We consider 15 art price indices namely: Contemporary, Drawings, France, Global index (Euro), Global index (USD), Modern art, Nineteenth century, Old Masters, Paintings, Photographies, Postwar, Prints, Sculptures, UK and US. We use quarterly data from 1998:1 to 2015: 1. We employ both standard and non-parametric single and joint variance ratio tests while accounting for small sample bias through the use of the wild bootstrapping. We show that the majority of the art market price indices are inefficient with the exception of the Old Masters that consistently prove efficient under both individual and joint tests. Also we cannot reject the null hypothesis of random walk or martingale for Contemporary, US and UK indices albeit not as strong as Old Masters suggesting that these three art indices are less predictable. Our results imply that future price movements in Old Masters, and to some extent in Contemporary, US and UK indices are determined entirely by information not contained in the price series whereas the remainder of the indices can be predicted using price information. However, confronting the data with both linear and nonlinear long memory models, we observe the following series can have unit roots: Paints, Prints, Photographies, Nineteenth century, Modern Art, US, France and Drawings and their markets are therefore efficient to a certain point. Post war, Sculpture, Drawings, France, Contemporary and France have values of the fractional parameter *d* significantly different from 0 and 1. The US and Contemporary art markets are indisputably efficient based on both long memory and variance ratio tests.

Keywords: Art market, market efficiency, variance ratio tests, random walk, martingale, non-parametric

JEL Classification: C14, G14, G15

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1. Introduction

Investors often seek alternative assets and sophisticated solutions to acquire high returns while minimizing risk, especially when faced with under-performing portfolios. Art indices is seen as a good investment option or strategy given the view that average annual returns has been as high as 10% for a very long period (Munteanu and Pece, 2015). Campbell (2008) also examines the financial gains that artworks have exhibited historically while including transaction costs, and show that arts offer diversification benefits as an investment portfolio given its low correlation with other asset classes. Also optimal portfolio allocations using historical returns make a case for investors to consider art as an attractive, albeit small addition to their investment strategy. Louargand and McDaniel (1991), Ashenfelter and Graddy (2003), after examining the auction market, argue that the increase in liquidity, better information in art auction catalogues, globalization, access to financing options, and the increase of participation in the auction markets are among the reasons that the art market has become more efficient. This suggests that art prices should be unpredictable or random (Louargand and McDaniel 1991). Further estimates based on hedonic price index of oil paintings, watercolors and drawings by Renneboog and Spaenjers (2013) show that between 1982 and 2007, art prices rose 3.97% annually thus outperforming returns from physical and financial assets: commodities (3.3%), gold (2.35%), US real estate (1.06%) and T-Bills (1.39%). This view is challenged recently by Korteweg et al. (2013) who argue that investors tend to overestimate the returns and underestimate the risks involved in selling and buying art work. Korteweg et al. (2015) show that average annual return for art investment decline from 10% to 6.2% making it less appealing than stock (10.95%), corporate bonds (8.94%) and commodities (10.21%).

Although there are many studies on different aspects of the art market, there is a dearth of studies on the art market efficiency as is evidenced in the literature section below. Under the weak-form efficiency, the returns are purely unpredictable and no investors are able to make abnormal profit consistently over time by exploiting past price information (Fama, 1970). Therefore, the understanding of the efficiency or otherwise of the art market may be useful to investors in their portfolio diversification decisions and risk management. In this study we contribute by investigating the art market efficiency (i.e. return predictability) for the recent period 1998 to 2015. We consider conventional and non-parametric variance ratio tests for both individual and multiple tests namely Lo and Mackinlay (1988), Wright's (2000) ranks and signs tests, Chow and Dennings (1993) and

Belaire-Franch and Contreras (2004) joint tests. Our choice of variance ratio tests instead of the unit root tests as in Çevik et al. (2013) and related papers is motivated by the fact that the former is a volatility-based test. This feature is important given that most asset returns often exhibit volatility and deviations from normality (Belaire-Franch et al., 2007). Hence tests such as the variance ratio tests which are robust to heteroskedasticity and non-normality become important. Earlier, Poterba and Summers (1988) investigate the power of different random walk tests and found that variance ratios are among the most powerful tests and it has even more power than the Fama and French (1988) regression based procedure. In addition, studies like that of Summers (1986), Poterba and Summers (1988), Cochrane (1988), Fama and French (1988), Lo and MacKinlay (1988), Liu and He (1991), Erdös and Ormos (2010) and Mobarek and Fiorante (2014) have argued that unit root tests have very low power against stationary alternatives and it is difficult to reject a false null hypothesis of random walk.[1] Note that from the perspective of data, our paper is closest to that of Çevik et al. (2013), as we use the same set of indices, but an updated period until 2015 and employ variance ratio tests rather than unit root tests. However, unlike a recent and closely related study on art market efficiency by Munteanu and Pece (2015) who, using our approach of the variance ratio tests, focused on four major auction houses – Sotheby's, Turners Auctions Ltd, Mallett PLC and Mowbray Collectables- we, as in Çevik et al. (2013), study a wider set of art indices including Contemporary, Drawings, France, Global (Euro), Global (USD), Modern art, Nineteenth century, Old Masters, Paintings, Photographies, Postwar, Prints, Sculptures, UK and US indices. Finally, for the conventional variance ratio tests which ordinarily use asymptotic normal probabilities to evaluate statistical significance, we use Kim's (2006) wild bootstrap p-values to guard against small sample bias. In addition to the variance ratio tests, we also analyze the efficiency of the art market using fractional techniques based on long memory models. These models are able to account for long memory behavior and nonlinearities evidenced in most financial and economic time series.

The remainder of the paper is organized as follows: Section 2 reviews the literature, while Section 3 discusses the data and the methodology. Section 4 presents the results, and Section 5 concludes.

[1] Unit root tests most commonly employed in the literature (Dickey and Fuller, 1979; Phillips and Perron, 1988; Kwiatkowski et al., 1992; etc.) have very low power against trend-stationarity (DeJong, Nankervis, Savin and Whiteman, 1992), structural breaks (Perron, 1989; Campbell and Perron, 1991), regime-switching (Nelson , Piger and Zivot, 2001), or fractional integration (Diebold and Rudebusch, 1991; Hassler and Wolters, 1995; Lee and Schmidt, 1996).

2. Literature Review

Many time series variables exhibit trend behavior and they are relevant to the study of market efficiency. According to Fama (1970), to be efficient, an asset should follow a random walk. In this case, the direction and prices of art should not be predictable. Investors must not profit from information already available. A market is said to be efficient if prices fully reflect all available information. In an efficient market, future returns are unpredictable. Fama (1970) draws a distinction between three forms of efficiency depending on the information set considered. In the weak efficiency form, only past and current prices are considered. In a weakly efficient market the current return is unrelated to past returns. The semi-strong form asserts that prices reflect all publicly available information. Lastly, the strong form states that prices reflect all available information, both public and private. Market efficiency is key for investors since it gives them confidence in the fairness of market valuation.

One problem with doing reliable efficiency analysis is difficulties associated with computing relevant series of art returns. Frey and Eichenberger (1995) argue that art market efficiency is impossible to test because of data limitations. However, some studies have attempted to examine this. For instance, David, et al. (2013) use an art index developed by Renneboog and Spaenjers (2013). They propose tests for weak efficiency of the art market based on time-series analysis. Their results show that art returns exhibit highly auto-regressive dynamics. This implies that weak efficiency is rejected.

Erdös and Ormos (2010) conduct variance ratio tests based on non-parametric methods to detect the size of the random walk component of the US art auction prices. They find that the past 134 years of US art prices exhibit large transitory component (72%) and consequently, the random walk hypothesis does not hold. They detect structural breakpoints and find that the random walk hypothesis and the weak-form efficiency of US art market cannot be rejected at least for the past 64 years. Applying the Cochrane (1988) variance ratio estimation, they find that the random walk hypothesis can be rejected for the whole sample period. However, if they exclude the first 60 years from the sample, the random walk hypothesis cannot be rejected.

Çevik et al. (2013) examine the time series properties of art price indices in the global art market and in its various segments. Their results indicate that the overall global art market (in USD) is found to be a stationary process at 1 % level. The price indices for the Sculptures, Photographs, Old Masters, Contemporary, Paintings and Prints are stationary, hence providing evidence of mean-reversion. The price indices for Drawings and Nineteenth century were found to be non-stationary and hence according to the authors are in some sense closest to exhibiting at least weak-form efficiency.

Munteanu and Pece (2015) investigate the returns obtained by auction houses. They test the market efficiency of the most influential auction house as a signal for art market robustness. They focus on how investors use information regarding the activity of four major auction houses – Sotheby's, Turners Auctions Ltd, Mallett PLC and Mowbray Collectables - and how this information is reflected in the stock price. They conduct several tests of market efficiency (in particular, Automatic Variance Ratio test, Joint Wright test, and the Lo and MacKinlay test). The results indicate that while some stocks exhibit market efficiency, others present a slow assimilation of information in the stock price and hence past information can be used to make predictions.

Frey and Pommerehne (1989) find that real art returns do not follow a normal distribution and conclude that prices seem not to follow a pure random process employing data for the period 1635-1949. This is contrary to the results of Baumol's (1986) for the period 1652-1961. Poterba and Summer (1988) show that the random walk tests have low power if only a short span of data is available. Pesando (1993) investigates the prints market over the 1977–1992 period and finds that excess returns are autocorrelated positively for the one-year lag and negatively for the two-year lag.

We consider conventional and non-parametric variance ratio tests for both individual and multiple tests namely ranks and signs tests and joint tests. The choice of variance ratio tests rather than the often used unit root tests is motivated by the fact that the former is a volatility-based test. The variance ratio test is robust to heteroskedasticity and non-normality becomes important. Furthermore, the power of the variance ratios has greater power than regression based procedure. Also unit root tests have very low power against stationary alternatives and it is difficult to reject a false null hypothesis of a random walk.

With respect to the contribution of our paper, our work is closest to that of Çevik et al. (2013) who conduct unit root tests on the level of art indices. We use the same set of indices, but we use an updated period until 2015 and employ variance ratio tests rather than unit root tests. Munteanu and Pece (2015) employ art return series on four major auction houses – Sotheby's, Turners Auctions Ltd, Mallett PLC and Mowbray Collectables- and employ variance ratio tests. We study a much wider set of art indices. Furthermore, for the conventional variance ratio tests which ordinarily use asymptotic normal probabilities to evaluate statistical significance, we use Kim (2006) wild bootstrap p-values to guard against small sample bias. In addition to the variance ratio tests, we also analyze the efficiency of the art market using fractional techniques based on long memory models. These models are able to account for long memory behavior and nonlinearities evidenced in most financial and economic time series. By allowing for fractional degrees of differentiation we allow for a higher degree of flexibility in the dynamic specification of the data, including as particular cases of interest the stationary I(0) and the nonstationary I(1) cases.

3. Data and Empirical Models

This study uses quarterly data spanning 1998:1 to 2015:1 on 15 art market price indices returns namely Contemporary, Drawings, France, Global index (Euro), Global index (USD), Modern art, Nineteenth century, Old Masters, Paintings, Photographies, Postwar, Prints, Sculptures, UK and US indices. The data is obtained from ARTPRICE, available for download from: http://www.artprice.com. The art market price indices are calculated by the Artprice™, a company specializing in art market information and data services. A repeat-sales model drawn from a database of over 27 million auction records from over 3,600 auction houses around the world were used in calculating the indices. The availability of comprehensive dataset and at quarterly frequency makes the data base maintained by Artprice™ appealing. All indices are transformed to their natural logarithms. The log of these variables are plotted and presented in Figure 1. It can be observed that the majority of the art indices show some level of volatility while some show upward trend. Overall, the series do not appear to exhibit clear random walk behavior. However, this will be proved by formal tests in the next section.

[Insert Figure 1 about here]

We also present in Table 1 the descriptive statistics for the art returns. With the exception of Old Masters and Nineteenth century, the rest of the returns have positive mean. The volatility of the series as evidenced by the standard deviations are relatively high with the Global index (USD) exhibiting the highest volatility while France exhibited the lowest. Majority of the indices are negatively skewed with most having kurtosis of approximately 4 which is slightly above the normal kurtosis of 3. The formal test of normality, the Jarque-Bera test, rejects the null of normality for four of the series namely Paintings and Prints at 5% level and Modern art and US return at 10% level. With respect to the LM test, the null of no ARCH effect is rejected for all series showing there is strong evidence of conditional heteroskedasticity in all the art return series.

[Insert Table 1 about here]

Variance ratio tests are designed to test for type-1 random walk (RW1), which assumes homoscedastic increments as well as type-3 random walk (RW3) if the increments are assume to be subject to heteroskedasticity (Campbell et al. 1997). The paper employs the different variance ratio tests, namely the Lo and Mackinlay (1988) test, Wright (2000) non-parametric tests, Chow and Denning (1993) and the Belaire-Franch and Contreras (2004) joint tests.

Lo and MacKinlay (1988) test

Lo and MacKinlay (1988) proposed a single variance ratio test which uses the fact that if the price of a series (e.g. (art index) at time, y_t, follows a random walk or martingale, then the increments are said to be serially uncorrelated and the variance of those increments should increase linearly in the sampling intervals,

$$y_t = y_{t-1} + \mu + \varepsilon_t,\ \varepsilon_t \sim \text{IID} \quad \text{N}(0, \sigma^2) \quad (1)$$

If the art index price is mean-reverting, its return is predictable ex ante in the form of a systematic pattern in its dependence on past prices and hence the market is not weak-form efficient. On the other hand, if the art index price follows a random work or martingale, the return is unpredictable from past price information and hence the market is efficient. For sample size $t = 1,\ldots T$, the variance ratio begins with the formulation (Wright 2000; Hoque et al., 2007; Charles and Darné, 2009; Mobarek and Fiorante, 2014):

$$VR(y;q) = \left\{ \frac{1}{Tq} \sum_{t=q}^{T} (y_t + y_{t-1} + \ldots + y_{t-q+1} - q\hat{\mu})^2 \right\} \div \left\{ \frac{1}{T} \sum_{t=1}^{T} (y_t - \hat{\mu})^2 \right\}, \quad (2)$$

where q is any positive integer. $\hat{\mu} = T^{-1}\sum_{t-1}^{T} y_t$. This is an estimator for the unknown population VR, denoted as $V(q)$, which is the ratio of $1/q$ times the variance of the q-period return to the variance of the one-period return. If y_t is independent and identically distributed (i.i.d), and under the assumption of homoscedastic increment, Lo and Mackinlay (1988) single variance ratio tests the null hypothesis that $\mathrm{H}_0 : V(q) = 1$, the test statistic:

$$M_1(y;q) = (VR(y;q) - 1)\left(\frac{2(2q-1)(q-1)}{3qT}\right)^{-1/2} , \tag{3}$$

follows a standard normal distribution asymptotically. To accommodate y_t's exhibiting conditional heteroskedasticity, Lo and Mackinlay (1988) proposed a heteroskedasticity robust test statistic

$$M_2(y;q) = (VR(y;q) - 1)\left(\sum_{j=1}^{q-1}\left[\frac{2(q-j)}{q}\right]^2 \delta_j\right)^{-1/2} , \tag{4}$$

also follows the standard normal distribution asymptotically under the null hypothesis that $V(q) = 1$, where

$$\delta_j = \left\{\sum_{t=j+1}^{T}(y_t - \hat{\mu})^2 (y_{t-j} - \hat{\mu})^2\right\} \div \left\{\left[\sum_{t=1}^{T}(y_t - \hat{\mu})^2\right]^2\right\} .$$

The M_2 test is applicable to y_t's generated from a martingale difference time series. The usual decision rule for standard normal distribution applies to M_1 and M_2.

Wright's* (2000) *rank-based and sign-based VR tests

The standard VR tests such as the Lo and Mackinlay (1988) tests are asymptotic tests whose sampling distribution are approximated based on their limiting distributions. The problem with this is that they are biased (severe size distortions and low power) and right-skewed in finite samples, resulting in misleading statistical inference (Lo and Mackinlay, 1989). This is especially true when the sample size is not large enough to justify the asymptotic approximations. Based on this Wright (2000) proposed non-parametric alternatives to the standard asymptotic VR tests using ranks and signs. Wright's (2000) tests have two main advantage over other single and multiple variance ratio tests: first, it is very likely to calculate exact sampling distribution, hence there is no need to resort to asymptotic approximation, and second, it may be more powerful than the conventional tests when the distribution is highly non-normal (Wright, 2000). The tests based on ranks are exact under the i.i.d assumption, whereas the tests based on signs are exact even under conditional heteroskedasticity. Moreover, the rank-based tests display low size distortion under conditional heteroskedasticity (Wright, 2000).

Given T observations of first differences (returns) of a variable, $\{y_1,..., y_T\}$ and let $r(y_t)$ be the rank of y_t among $y_1, y_2,..., y_T$. Under the null hypothesis that y_t is generated from an i.i.d. sequence, $r(y_t)$ is a random permutation of the numbers $1,...,T$ with equal probability. Wright (2000) suggests two rank-based test statistics R_1 and R_2 defined as

$$R_1(q) = \left(\frac{(Tq)^{-1} \sum_{t=q}^{T} (r_{1t} + r_{1t-1} + ... + r_{1t-q+1})^2}{T^{-1} \sum_{t=1}^{T} r_{1t}^2} - 1 \right) \left(\frac{2(2q-1)(q-1)}{3qT} \right)^{-1/2} \tag{5}$$

,

and

$$R_2(q) = \left(\frac{(Tq)^{-1} \sum_{t=q}^{T} (r_{2t} + r_{2t-1} + ... + r_{2t-q+1})^2}{T^{-1} \sum_{t=1}^{T} r_{2t}^2} - 1 \right) \left(\frac{2(2q-1)(q-1)}{3qT} \right)^{-1/2} \tag{6}$$

where the standardized ranks are given by

$$r_{1t} = \frac{r(y_t) - 0.5(T+1)}{\sqrt{((T-1)(T+1)/12}}$$

and

$$r_{2t} = \Phi^{-1} \frac{r(y_t)}{T+1},$$

ϕ^{-1} is the inverse of the standard normal cumulative distribution function. The R_1 and R_2 test statistics share the same exact sampling distribution and their critical values can be obtained by simulating their exact distributions.

Similarly Wright (2000) suggests two tests statistics S_1 and S_2 based on the signs of first differences (returns).[2] Under the null hypothesis that y_t is a martingale difference sequence whose unconditional mean is zero and that s_t is an i.i.d sequence with mean 0 ($\bar{\mu}=0$) and variance equal to 1, which takes the value of 1 or otherwise -1 with equal probability of 0.5, S_1 is given by

$$S_1(q)=\left(\frac{(Tq)^{-1}\sum_{t=q}^{T}(s_t+s_{t-1}+\ldots+s_{t-q+1})^2}{T^{-1}\sum_{t=1}^{T}s_t^2}-1\right)\left(\frac{2(2q-1)(q-1)}{3qT}\right)^{-1/2} \qquad (7)$$

Multiple variance ratio tests

Lo and MacKinlay (1988) and Wright (2000) tests are both single variance ratio tests. The single variance ratio may not be completely adequate for testing the random walk hypothesis, as it is useful for testing only the individual variance ratios for a specific interval q. It is noted that testing with different q values would lead to over rejection of the null hypothesis. Chow and Denning (1993) noted that this sequential procedure leads to size distortion. To overcome this problem Chow and Denning (1993) provide a multiple VR test where the variance ratio of all observation intervals, $q's$ need to be simultaneously equal to 1. The joint null hypothesis is $\mathrm{H}_0: V(q_i)=1$ for $i=1,\ldots,m$ against the alternative that $\mathrm{H}_1: V(q_i)\neq 1$ for some holding period q_i. The joint test statistic for the homoskedastic assumption associated with the Lo ad Mackinlay (1988) test is given as

$$MV_1=\sqrt{T}\max_{1\le i\le m}\left|M_1(y;q_i)\right| \qquad (9)$$

This is based on the idea that the decision with respect to the null hypothesis can be made based on the maximum absolute value of the individual VR statistics. The statistic follows the studentized maximum modulus distribution with m (number of variance ratios) and T (sample size) degrees of freedom and uses this critical values rather than the standard normal distribution. When T is large, the critical values can be computed based on the limiting distribution of the statistic. Similarly, the heteroskedasticity-robust version of the Chow-Denning (1993) test can be written as

[2] The test S_2 tests for a random walk with a drift. However, it should be noted that Wright's (2000) S_2 test is not considered here, as his Monte Carlo simulations clearly show that its size and power properties are quite inferior to those of S_1.

$$MV_2 = \sqrt{T} \max_{1 \le i \le m} \left| M_2(y; q_i) \right| \quad (10)$$

Belaire-Franch and Contreras (2004) also proposed multiple rank and sign VR tests by substituting Wright's (2000) rank and sign-based tests in the definition of the Chow and Denning (1993) multiple test procedure. The ranks-based and signs-based multiple variance ratio test statistics are defined as

$$CD_{(R_1)} = \max_{1 \le i \le m} \left| R_1(q_i) \right| \quad (11)$$

$$CD_{(R_2)} = \max_{1 \le i \le m} \left| R_2(q_i) \right| \quad (12)$$

and

$$CD_{(S_1)} = \max_{1 \le i \le m} \left| S_1(q_i) \right| \quad (13)$$

Similarly to Wright's (2000) individual tests, the ranks-based multiple VR tests are sensitive to deviations from the stronger i.i.d. assumptions. However, signs-based multiple VR tests are robust to conditional heteroskedasticity, although $CD_{(S_1)}$ is constructed under the additional assumption of zero drift value.[3]

Kim (2006) wild bootstrap test

Kim (2006) uses the wild bootstrap which is a resampling method that approximates the sampling distribution of the VR test statistic, and is applicable to data with unknown forms of conditional and unconditional heteroskedasticity (Mammen, 1993; Davidson and Flachaire, 2008; Mackinnon, 2002). The wild bootstrap could be applied to both the Lo and Mackinlay (1988) single variance tests and Chow and Denning (1993) multiple variance tests. We use the wild bootstrap instead of the asymptotic normal probabilities to evaluate the statistical significance of Lo and Mackinlay (1988) tests and its associated Chow-Denning joint tests. Usually a specific form of bootstrap error distribution is required to implement the wild bootstrap test. Kim (2006) recommends using the standard normal distribution since other choices such as the two-point distribution of Mammen (1993), and the Rademacher distribution discussed in Davidson and Flachaire (2008) produced qualitatively similar sample results. The unit root tests and the approaches presented above test the efficiency market hypothesis by using the random walk model. This is a very special case of the I(1) behaviour and, as earlier mentioned, fractional integration allows for a greater flexibility in the specification of the model.

[3] Belaire-Franch and Contreras (2004) showed that the rank-based tests are more powerful than the sign-based tests.

Fractional Integration Model

This methodology is based on the concept of long memory or long range dependence that is characterized because observations which are quite distant in time still present a degree of dependence between them. We focus on fractional integration which is probably the most widely used form of long memory processes.

A process $\{x_t, t = 0, \pm 1, \ldots\}$ is said to be integrated of order d, and denoted by $x_t \approx I(d)$, if after taking d differences, the process becomes I(0) (or integrated of order 0), which is defined as a covariance stationary process where the infinite sum of the autocovariances is finite. In other word, x_t is I(d) if:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \ldots, \tag{14}$$

with $x_t = 0, t \leq 0$, L is the lag operator (i.e., $Lx_t = x_{t-1}$), and u_t is $I(0)$. Fractional integration takes place when d is a fractional value. Note that the polynomial on the left-hand-side of equation (14) can be expanded, for all real d, as

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - d L + \frac{d(d-1)}{2} L^2 - \ldots, \tag{15}$$

implying that x_t in (4) can be written as:

$$x_t = d x_{t-1} - \frac{d(d-1)}{2} x_{t-2} + \ldots + u_t .$$

Thus, the higher the d is, the higher the level of dependence is between the observations. The literature has focuses during many years on integer degrees of differentiation, basically, d = 0 (in case of stationary I(0) processes) or d = 1 (in case of unit roots or nonstationary processes), however, by allowing d to be any real value, we allow for a greater degree of flexibility in the dynamic specification of the series.

There exist many methods for estimating and testing the (fractional) differencing parameter d. They can be specified in the time domain or in the frequency domain, and they can be parametric, where the model is completely specified and d is merely an additional parameter in the model or semiparametric. In this paper we focus on parametric methods and estimate d by means of a Whittle function in the frequency domain (Dahlhaus, 1989), though several other maximum likelihood methods (Sowell, 1992; Beran, 1995) were also computed obtained practically the same results.

In the results presented in the following section, we consider a model of the following form,

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad (1-L)^d x_t = u_t, \quad t = 1, 2, \ldots, \tag{16}$$

where y_t is the observed series, and under the assumption that the error term u_t is a white noise process, and we present the results for the three standard cases examined in the literature: the case of no deterministic terms (i.e., $\beta_0 = \beta_1 = 0$ a priori in the above equation); an intercept (β_0 unknown and $\beta_1 = 0$), and with both an intercept and a linear time trend (both β_0 and β_1 unknown).[4]

Finally, it has been argued in recent years that fractional integration may be a spurious phenomenon caused by the presence of structural breaks or nonlinearities in the data (see, e.g., Diebold and Inoue, 2001; Kapetanios and Shin, 2011; etc.). Because of that, the possibility of nonlinear deterministic trends is also taken into account by introducing the Chebyshev polynomials in time as in Cuestas and Gil-Alana (2016). This approach is preferred over the method proposed by Gil-Alana (2008), whereby the number of breaks and the break dates in the series are determined endogenously, obtained by minimising the residual sum of the squares at different break dates and different (possibly fractional) differencing parameters. The reasons are both intuitive and technical. First, from an intuitive point of view, since, we are using low-frequency data, structural breaks should ideally be modelled in a smooth rather than an abrupt fashion. And second, from the technical side, our small sample size does not allow us to apply the method proposed by Gil-Alana (2008). Thus, we consider, instead of the left hand side equation in (15), the following regression model

$$y_t = \sum_{i=0}^{m} \theta_i P_{iT}(t) + x_t, \qquad t = 1, 2, \ldots, \tag{17}$$

with m indicating the order of the Chebyshev polynomial, and x_t following an I(d) model of the form as in the second equation in (16). The Chebyshev time polynomials $P_{i,T}(t)$ in (17) are defined by:

$$P_{0,T}(t) = 1,$$

$$P_{i,T}(t) = \sqrt{2}\cos\left(i\pi(t-0.5)/T\right), \qquad t = 1, 2, \ldots, T; \quad i = 1, 2, \ldots \tag{18}$$

See Hamming (1973) and Smyth (1998) for a detailed description of these polynomials. Bierens (1997) uses them in the context of unit root testing. According to Bierens (1997) and Tomasevic and Stanivuk (2009), it is possible to approximate highly non-linear trends with a rather low degree of polynomials. If $m = 0$ in (17), the model contains an intercept, if $m = 1$ it adds a linear trend, and if

[4] Note that the random walk model given by (1) is included here as a particular case with d = 1, $\beta_1 = \mu$ and $\beta_2 = 0$.

$m > 1$ the model becomes non-linear, and the higher m is the less linear the approximated deterministic component becomes.

4. Empirical Results

Variance Ratio Tests Results

We investigate the weak-form efficient market hypothesis by testing the random walk or martingale difference hypothesis using variance ratio tests. This paper employs the standard variance ratio test of Lo and MacKinlay, (1988, 1989), the non-parametric-based variance ratio test of Wright (2000) and the joint or multiple-variance ratio test of Chow and Denning (1993) and its modified version by Belaire-Franch and Contreras (2004). The holding periods $(q_i 's)$ considered are 2, 4, 8 and 16. This is consistent with Deo and Richardson (2003) who advocated for relatively short holding periods when testing for mean-reversion using the variance ratio tests. Table 2 presents the variance ratios for the individual tests. For the Lo and MacKinlay (1988, 1989) tests, we report the results based on both homoscedasticity (M1) and heteroskedasticity (M2) assumptions. The statistical significance of the tests are based on Kim's (2006) wild bootstrap probabilities to guard against small sample bias. The bootstrap p-values for both Lo and Mackinlay (1998) and Wright (2000) tests are computed using 10000 replications.[5]

[Insert Table 2 about here]

Starting with the Contemporary art returns, we observe that the null hypothesis is rejected at 1% for period 2 under the Lo and Mackinlay's (1988) homoscedasticity version of the test. At longer holding periods (4, 8 and 16) however, the null could not be rejected. Similar result is observed under the heteroskedastic assumption with the exception that here the null is rejected at 10% for period 4. Wright's (2000) tests produce similar results for Contemporary with the null being rejected for period 2 in the rank and rank-score test while it was not rejected under the sign-based test. The null is rejected mainly at periods 2 and 4 for Drawings, Nineteenth century, Photographies and Sculptures. For France and Prints the null is rejected essentially at all periods except at period 16. The null is rejected for Global index (USD) for periods 8 in all cases and 16 in addition for M2 and

[5] To conserve space, the *p*-values are not presented here.

S1 tests. For UK, rejections are observed for only period 2 across all tests while the null cannot be rejected at any level and under any of the tests for Old Masters. US had no rejections at period 16 for Lo and Mackinlay (tests) and at basically periods 8 and 16 for Wright (2000) tests. Overall, at period 2, all individual tests consistently reject the null hypothesis for all returns except the Old Masters. Beyond period 2, results are mixed.

[Insert Table 3 about here]

Turning to the joint tests as presented in Table 3, we observe that except for Contemporary, Old Masters, UK and US, all the tests reject the null of random walk or martingale for all the art returns. Whereas the null for UK is rejected by CD(M2), the null for Contemporary and US are rejected by CD(S1) joint tests. However, there is overwhelming evidence of weak-form efficiency for Old Masters consistent with the results from the individual tests. Thus, we can strongly conclude that Old Masters is not predictable. The conclusion is not as strong for Contemporary, UK and US art returns and essentially non-existent for the rest of the returns meaning that these other markets can be predicted based on past prices and hence are not efficient.

Fractional Integration Model Results

Tables 4 refers to the estimates of the differencing parameter d (and their corresponding 95% confidence intervals) for the art returns, in the linear model given by (16) for the three standard cases of no regressors, an intercept, and an intercept with a linear trend. We observe that a linear time trend seems to be required in case of the Global index (in USD), while in the remaining cases, no deterministic terms are required. All the estimated values of d are below 1; however, we observe substantial differences from one case to another. Starting with those with the highest degrees of dependence, we observe that the unit root null hypothesis (i.e., d = 1) cannot be rejected in the cases of Prints (d = 0.98), Nineteenth century (0.67), Paintings (0.66), US (0.52), Modern art (0.51) and Photographies (0.32).

Another group of variables are those where the estimated value of d is significantly different from 0 and 1. Here, we include the following series: Post war (0.40), Sculptures (0.39), Drawings (0.37), France (0.36), and Contemporary (0.29); There are two series where the I(0) hypothesis cannot be rejected: Photographies (0.32), UK (0.17) and Old Masters (-0.12); and finally the Global index (in

USD) displayed anti-persistence (i.e, $d < 0$). Thus, we observe a large degree of heterogeneity in the results presented.

[Insert Table 4 about here]

Nevertheless, the possibility of nonlinear trends is also taking into account. Tables 5 reports the estimates of d in (14) along with the non-linear coefficients, θ_i (for m = 1 to 4) in (17) and their corresponding t-values. Note that θ_3 and θ_4 refer to the nonlinear Chebyshev coefficients in time. We observe only two return series that display some degree of nonlinearity and they are Old masters and the Global index (USD). If we look at the degrees of integration, though quantitatively we observe some differences, qualitatively they are very similar to those reported in Table 4. The null of unit root hypothesis (i.e $d=1$) cannot be rejected for Paintings, Prints, Photographies, Drawings, Nineteenth century, Modern art and US. The estimated value of d for France is significantly different from 0 and 1. The null of I(0) hypothesis cannot be rejected for the following series: Sculptures, Photographies, Old Masters, Modern art, Post war, Contemporary and UK. The Global index (USD) exhibits again an anti-persistent behavior.[6]

Our results may be compared to that of Çevik et al. (2013) who used the same indices though with a different methodology. Their unit root tests show that Sculptures, Photographies, Old Masters, Contemporary, Paintings and Prints are stationary while Drawings and Nineteenth century are non-stationary and hence are weak-form efficient. These findings contrast with ours. However, we note that the unit root tests have lower power than both the variance ratio tests and tests based on the long memory approach.

[Insert Table 5 about here]

5. Conclusions

This paper examines the weak-form efficiency hypothesis for 15 art price indices including the Contemporary, Drawings, France, Global index (Euro), Global index (USD), Modern art,

[6] Note that, the results for Global index based on the Euro is, not surprisingly, the same as the Global index for USD in general. The details of these results are available upon request from the authors. Note that, the results for Global index based on the Euro is, not surprisingly, the same as the Global index for USD in general. The details of these results are available upon request from the authors.

Nineteenth century, Old Masters, Paintings, Photographies, Postwar, Prints, Sculptures, UK and US. We use quarterly data from 1998:1 to 2015:1.

The understanding of efficiency for the art market is useful to investors in terms of their portfolio diversification decisions and risk management. In this study we contribute by investigating the art market efficiency (i.e. return predictability) for the recent period 1998 to 2015. We consider conventional and non-parametric variance ratio tests for both individual and multiple tests namely ranks and signs tests and joint tests. The choice of variance ratio tests rather than the often used unit root tests is motivated by the fact that the former is a volatility-based test. The variance ratio test is robust to heteroskedasticity and non-normality becomes important. Furthermore, the power of the variance ratios has greater power than regression based procedures. Also unit root tests have very low power against stationary alternatives and it is difficult to reject a false null hypothesis of a random walk.

Our paper is closest to that of Çevik et al. (2013) who conduct unit root tests on the level of art indices. We use the same set of indices, but we use an updated period until 2015 and employ variance ratio tests rather than unit root tests. Munteanu and Pece (2015) employ art return series on four major auction houses – Sotheby's, Turners Auctions Ltd, Mallett PLC and Mowbray Collectables- and employ variance ratio tests. We study a much wider set of art indices. Finally, for the conventional variance ratio tests which ordinarily use asymptotic normal probabilities to evaluate statistical significance, we use Kim's (2006) wild bootstrap p-values to guard against small sample bias. In addition to the variance ratio tests, we also analyze the efficiency of the art market using fractional techniques based on long memory models. These models are able to account for long memory behavior and nonlinearities evidenced in most financial and economic time series.

Our results indicate that the random walk or martingale hypothesis for returns is tested using both individual and multiple variance ratio tests and holding periods of 2, 4, 8 and 16. Results based on the individual tests shows that the null hypothesis is rejected for all the returns at period 2. However, beyond period 2, the results are inconclusive as the hypothesis is rejected at some periods while it is not at other periods. The joints tests show consistent non-rejection of the null hypothesis for Old Masters while the evidence is somewhat weak for Contemporary, US and UK. For the rest of the returns, the joint tests strongly reject the null hypothesis of random walk or martingale. Based on the joint tests we can conclude that Old Masters, Contemporary, US and UK art markets are efficient and hence cannot be predicted based on past price information; thus making it difficult for investors to make excess returns. In the second part of the paper we enrich the analysis by using fractional integration which includes the I(0) and I(1) hypotheses as particular cases of interest.

Results based on the linear long memory model for the return series shows that the null hypothesis of unit root cannot be rejected in the cases of Prints, Nineteenth Century, Paintings, US in USD, Modern Art and Photographies. The estimated value of d for Post war, Sculptures, Drawings, France, and Contemporary are significantly different from 0 and 1. Results from the nonlinear counterpart show that the null of unit root hypothesis cannot be rejected for Paintings, Prints, Photographies, Drawings, Nineteenth century, Modern art and US. Here the estimated value of d for France is significantly different from 0 and 1. The rest of the series have short memory. Overall, only the US and perhaps Contemporary are unequivocally identified as efficient based on both the variance ratio tests and long memory models.

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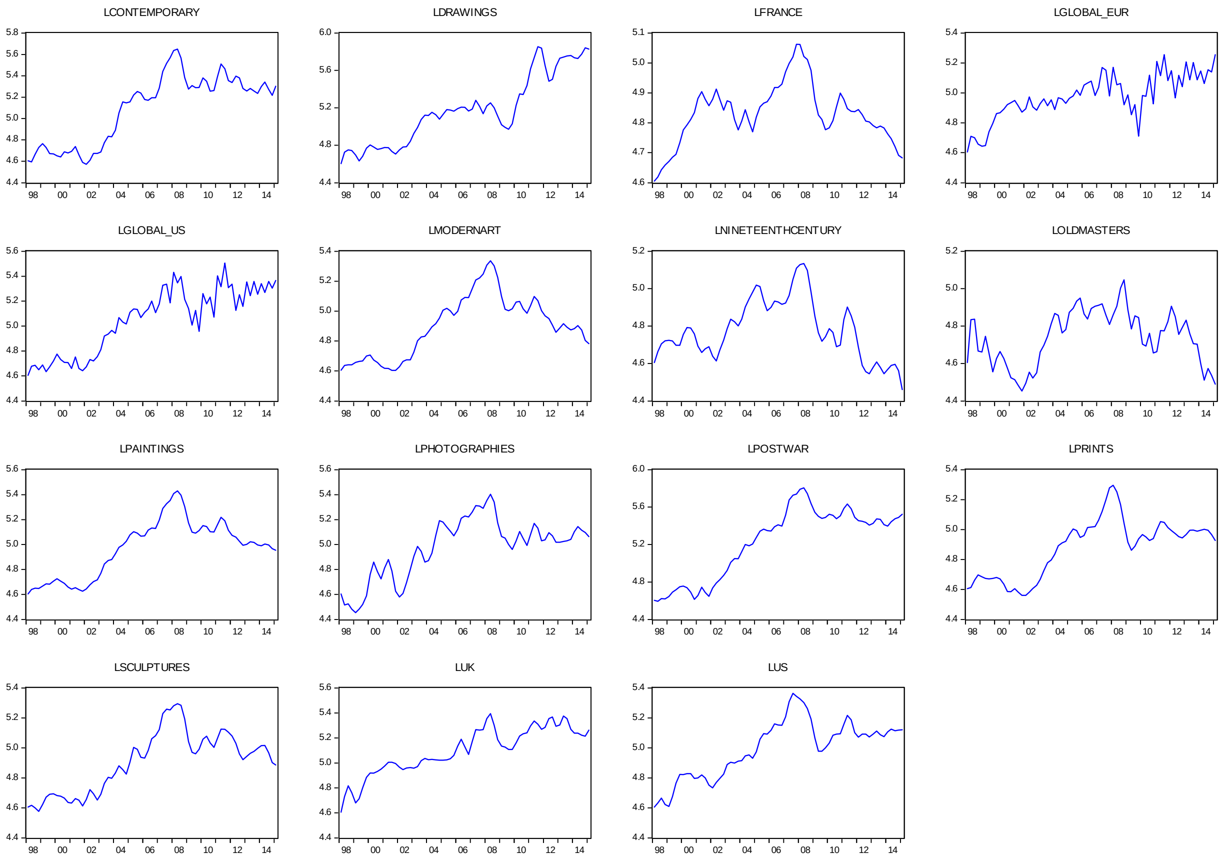

Figure 1: Log of art indices

Table 1: Descriptive statistics of the art return series

Series	Mean	Median	Max	Min	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	LM
Contemporary	1.026	0.516	16.079	-18.110	6.585	-0.158	3.192	0.388	3.614*
Drawings	1.800	1.251	19.542	-18.722	6.978	-0.130	3.851	2.242	7.360†
France	0.115	0.523	4.964	-9.863	3.005	-0.545	3.264	3.558	3.139*
Global index (Euro)	0.956	1.356	28.387	-21.084	10.265	0.108	3.218	0.268	8.656†
Global index (USD)	1.117	2.064	33.216	-21.120	10.981	0.424	3.568	2.947	2.983*
Modern art	0.262	0.552	7.666	-12.664	3.949	-0.614	3.659	5.508*	10.798†
Nineteenth century	-0.213	0.425	13.603	-12.864	5.247	-0.297	2.950	1.006	6.898†
Old Masters	-0.170	0.417	22.873	-16.987	7.269	0.034	3.397	0.460	16.309†
Paintings	0.514	0.851	9.309	-12.992	3.902	-0.678	4.387	10.653†	15.774†
Photographies	0.673	0.592	17.142	-16.871	7.008	-0.112	2.735	0.342	3.662*
Postwar	1.352	1.769	16.143	-10.650	5.082	-0.014	3.254	0.185	8.861†
Prints	0.473	0.643	7.977	-12.654	4.061	-0.876	4.579	15.763†	28.706†
Sculptures	0.415	-0.023	10.625	-15.138	4.655	-0.350	3.734	2.915	8.824†
UK	0.968	0.684	12.663	-11.623	4.917	-0.094	3.070	0.114	8.880†
US	0.758	0.501	9.993	-12.539	4.142	-0.451	3.951	4.866*	14.863†

† and * indicate significance at 5% and 10% level respectively.

Table 2: Individual variance ratio tests of return series

Test	q	Contemporary	Drawings	France (Euro)	Global index (Euro)	Global index (USD)	Modern art	Nineteenth century	Old Masters	Paintings	Photo-graphies
	2	3.327†	3.753†	3.820†	-5.460†	-4.833†	4.603†	4.489†	0.177	5.169†	3.448†
	4	1.629	1.961**	3.187†	-3.356†	-2.926†	3.459†	2.905†	-1.272	4.736†	1.027
	8	0.677	0.386	2.612†	-2.252**	-2.095**	2.795†	0.942	-0.846	3.567†	-0.357
M1	16	-0.054	-0.834	0.309	-1.706**	-1.616**	2.097†	-0.038	-1.029	2.244†	-0.575
	2	2.837†	2.942†	3.170†	-4.086†	-3.938†	3.327†	3.550†	0.263	3.413†	3.147†
	4	1.687*	1.930**	2.838†	-2.536†	-2.319**	2.835†	2.496†	-1.326	3.595†	1.356
	8	1.005	1.076	2.444†	-1.650*	-1.517*	2.553†	0.893	-0.855	3.146†	0.238
M2	16	0.681	0.567	0.292	-1.275	-1.072	2.164**	-0.022	-1.133	2.467**	0.155
	2	2.787†	3.249†	3.626†	-5.462†	0.391†	3.969†	4.401†	0.637	4.724†	3.076†
	4	1.206	1.567	2.722†	-3.456†	0.326†	2.702†	2.549†	-0.799	4.309†	0.618
	8	0.604	0.393	2.451†	-2.300†	0.324**	2.527†	0.853	-0.570	3.703†	-0.561
R1	16	0.015	-0.540	0.334	-1.706†	0.279	2.099†	0.010	-0.929	2.442†	-0.573
	2	3.232†	3.618†	3.423†	-5.552†	-5.022†	4.093†	4.539†	0.332	4.918†	3.466†
	4	1.579	1.946**	2.556†	-3.492†	-2.971†	2.849†	2.938†	-1.129	4.461†	1.204
	8	0.801	0.555	2.157†	-2.323†	-1.885**	2.581†	0.932	-0.799	3.613†	-0.122
R2	16	0.069	-0.558	0.008	-1.696†	-1.351	2.065†	-0.126	-1.037	2.227†	-0.283
	2	0.970	1.940†	3.881†	-3.153†	-3.153†	3.395†	2.910†	1.455	2.910†	2.183**
	4	-0.130	-0.130	4.149†	-1.880**	-2.009**	2.658†	0.324	0.389	2.528†	-0.130
	8	-0.225	-0.287	3.833†	-0.881	-1.271	2.132†	-0.553	0.266	2.849†	-1.066
S1	16	-0.179	-0.413	1.398	-0.895	-1.316	1.357**	-0.882	-0.069	2.803†	-0.999

Note: †, ** and * represent rejection of the null hypothesis of random walk or martingale at 1%, 5% and 10% respectively. M1 relates to the Lo and Mackinlay (1988) homoskedasticity (i.e no bias correction) VR test while M2 relates to the case of heteroskedaticity. R1 is the Wright (2000) rank test, R2 is the rank score test and S1 is the sign-based test. q is the holding period.

Table 3: Joint or multiple variance ratio tests on return series

Series	CD(M1)	CD(M2)	CD(R1)	CD(R2)	CD(S1)
Contemporary	3.327†(0.003)	2.837**(0.012)	2.787†(0.008)	3.232†(0.001)	0.970(0.655)
Drawings	3.753†(0.003)	2.941†(0.014)	3.249†(0.003)	3.618†(0.001)	1.940*(0.067)
France (Euro)	3.820†(0.001)	3.170†(0.009)	3.626†(0.001)	3.423†(0.002)	4.149†(0.000)
Global index (Euro)	5.460†(0.000)	4.086†(0.000)	5.462†(0.000)	5.552†(0.000)	3.153†(0.003)
Global index (USD)	4.833†(0.000)	3.938†(0.003)	5.022†(0.000)	4.892†(0.000)	3.153†(0.003)
Modern art	4.603†(0.000)	3.327†(0.009)	3.969†(0.000)	4.093†(0.000)	3.395†(0.001)
Nineteenth century	4.489†(0.000)	3.550† (0.005)	4.401†(0.000)	4.539†(0.000)	2.910†(0.005)
Old Masters	1.272(0.530)	1.326(0.576)	0.929(0.728)	1.129(0.583)	1.455(0.286)
Paintings	5.169†(0.000)	3.595†(0.006)	4.724†(0.000)	4.918†(0.000)	2.910†(0.007)
Photographies	3.448†(0.004)	2.998†(0.010)	3.076†(0.004)	3.466†(0.000)	2.183**(0.045)
Post war	3.913†(0.000)	3.249†(0.007)	3.493†(0.001)	3.780†(0.000)	2.425**(0.025)
Prints	5.620†(0.000)	3.316† (0.009)	4.889†(0.000)	5.153†(0.000)	3.881†(0.001)
Sculptures	3.954†(0.001)	3.062† (0.009)	3.470†(0.001)	3.709†(0.001)	2.425**(0.021)
UK ((in GBP)	2.490*(0.071)	2.065(0.113)	2.641†(0.014)	2.616†(0.013)	2.183**(0.037)
US (USD)	4.437†(0.001)	3.219†(0.007)	3.376†(0.002)	3.932†(0.000)	1.698(0.139)

Note: †, ** and * represent rejection of the null hypothesis of random walk or martingale at 1%, 5% and 10% respectively. Bootstrapped p-values are in parenthesis. CD(M1) is the Chow and Denning (1993) joint test associated with Lo and Mackinlay (1988) homoskedasticicity version of the VR test, CD(M2) is associated with Lo and Mackinlay (1988) heteroskedasticicity version, CD(R1) is associated with Wright (2000) rank test, CD(R2) is associated with the rank-score test while CD(S1) is associated with the sign-based test.

Table 4: Estimates of d (and 95% confidence intervals) in the return series

Series	No regressors	An intercept	A linear time trend
Global index (USD)	-0.48 (-0.56, -0.35)	-0.51 (-0.62, -0.35)	**-0.55 (-0.68, -0.38)**
Paintings	**0.66 (0.35, 1.29)**	0.67 (0.35, 1.32)	0.68 (0.33, 1.32)
Prints	**0.98 (0.51, 1.74)**	0.98 (0.52, 1.76)	0.98 (0.52, 1.80)
Sculptures	**0.39 (0.10, 0.99)**	0.39 (0.10, 0.98)	0.37 (0.06, 0.98)
Photographies	**0.32 (-0.02, 1.03)**	0.32 (-0.02, 1.08)	0.32 (-0.09, 1.08)
Drawings	**0.37 (0.03, 0.96)**	0.40 (0.03, 1.16)	0.41 (0.02, 1.16)
Old Masters	**-0.12 (-0.29, 0.16)**	-0.12 (-0.29, 0.16)	-0.14 (-0.33, 0.19)
Nineteenth century	**0.67 (0.18, 1.56)**	0.80 (0.18, 1.68)	0.83 (0.18, 1.66)
Modern art	**0.51 (0.24, 1.11)**	0.51 (0.24, 1.10)	0.50 (0.21, 1.11)
Post war	**0.40 (0.17, 0.82)**	0.38 (0.16, 0.83)	0.38 (0.12, 0.83)
Contemporary	**0.29 (0.03, 0.88)**	0.29 (0.03, 0.87)	0.27 (0.00, 0.88)
US (in USD)	**0.52 (0.21, 1.08)**	0.51 (0.20, 1.08)	0.51 (0.17, 1.08)
UK (in GBP)	**0.17 (-0.22, 0.90)**	0.15 (-0.17, 0.92)	0.13 (-0.24, 0.92)
France (in Euro)	**0.36 (0.15, 0.92)**	0.35 (0.15, 0.70)	0.30 (0.06, 0.69)

In bold the significant models according to the deterministic terms. In parenthesis the 95% confidence bands.

Table 5: Estimates of d and nonlinear coefficients in the return series

Series	d	θ_1	θ_2	θ_3	θ_4
Global index (USD)	-0.72 (-0.89, -0.49)	1.103 (1.05)	**0.378 (2.10)**	-0.254 (-1.09)	**-0.765 (-2.65)**
Paintings	0.64 (0.20, 1.32)	1.770 (0.43)	1.486 (0.64)	-0.612 (-0.36)	-0.674 (-0.51)
Prints	0.97 (0.47, 1.65)	1.828 (0.14)	0.733 (0.10)	-0.603 (-0.16)	-0.829 (-0.32)
Sculptures	0.31 (-0.09, 0.98)	0.456 (0.27)	0.964 (0.82)	-0.816 (-0.79)	-0.478 (-0.51)
Photographies	0.26 (-0.23, 1.07)	0.348 (0.16)	0.714 (0.44)	-0.458 (-0.32)	-1.223 (-0.92)
Drawings	0.41 (0.02, 1.16)	2.467 (0.66)	0.277 (0.11)	0.535 (0.27)	0.260 (0.15)
Old Masters	-0.28 (-0.52, 0.11)	-0.164 (-0.55)	**0.827 (2.01)**	**-0.858 (-1.85)**	-0.527 (-1.04)
Nineteenth century	0.83 (0.10, 1.57)	2.664 (0.24)	2.237 (0.35)	-0.346 (-0.09)	-0.177 (-0.06)
Modern art	0.43 (-0.01, 1.09)	0.643 (0.31)	1.408 (1.09)	-0.779 (-0.72)	-0.600 (-0.65)
Post war	0.27 (-0.08, 0.76)	1.478 (0.95)	0.959 (0.85)	-0.729 (-0.72)	-1.407 (-1.54)
Contemporary	0.21 (-0.17, 0.87)	1.190 (0.70)	0.830 (0.63)	-0.873 (-0.73)	-1.025 (-0.91)
US (in USD)	0.49 (0.15, 1.07)	1.106 (0.39)	0.899 (0.53)	0.002 (0.02)	-0.491 (-0.43)
UK (in GBP)	0.12 (-0.27, 0.90)	1.053 (1.14)	0.818 (1.03)	0.231 (0.30)	0.532 (0.73)
France (in Euro)	0.30 (0.06, 0.69)	0.112 (0.11)	0.940 (1.31)	0.034 (0.05)	0.236 (0.41)

In bold, the significant coefficients at the 5% level.