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## TRENDS AND CYCLES IN HISTORICAL GOLD AND SILVER PRICES

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### Abstract

The study proposes an alternative modelling specification for the real prices of gold and silver that allows the long run trend and cyclical behaviour to be modelled simultaneously by incorporating two differencing parameter in a fractional integration framework. However, we also consider the separate cases of a standard  $I(d)$  process, with a pole or singularity at the zero frequency and a cyclical  $I(d)$  model that incorporates a single pole in the spectrum at a non-zero frequency. We use annual data spanning from 1833 to 2013 for gold and 1792 to 2013 for silver. Based on the more flexible model that permits a pole at both zero (trend) and non-zero (cycle) frequencies, we find that in general the estimates associated to the long run or zero frequency appear to be above 1 in case of gold and below 1 for silver, while the order of integration associated with the cyclical frequency is slightly above 0 in the majority of the cases in the two series. Further, higher orders of integration are associated to the long run component compared with the cyclical one. The implications for these findings are highlighted.

**Keywords:** Gold and Silver Prices, Cycles, Persistence, Long memory

**JEL Classification:** C22, Q02

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## **1. Introduction**

Commodity prices are generally known to be very volatile leading to uncertainty over future revenue and cost streams. This consequently inhibits planning, deters investment and hence acts as a drag on future growth and poverty reduction prospects (Blattman et al., 2007; Poelhekke and van der Ploeg, 2009). The recent global economic and financial crisis which led to significant increase in commodity prices between 2003 and 2008 has renewed interest in modelling commodity price behaviour. The real prices of energy and metals more than doubled in five years from 2003 to 2008, while the real price of food commodities increased 75% (Erten and Ocampo, 2013). When the global economic growth slowed down, this led to diminishing demand pressures on commodity prices. However, commodity prices have started to recover very fast, and this has been attributed to the rapid growth of the emerging markets and dramatic increase in worldwide demand for commodities (Rossen, 2014).

Given the swings in prices, the need to model the trend and cycle behaviour of metal prices in general and gold and silver prices in particular cannot be overstressed. They are important for both the producing and consumer countries and for both the private and government. For the producing countries especially in developing countries, export earnings from metals are often the main source of revenue for many governments. The revenues may either come from direct ownership of resource extraction companies or the tax revenues and royalties obtained from private firms (Harvey et al., 2012). For the consumer countries, they are a key input factor in many industries and hence drastic price increases can affect these industries negatively through higher input costs (Rossen, 2014). Therefore, the dynamics of commodity prices are of great importance for worldwide economic activity (Labys, 2006) and as such fluctuations in their prices may have a major impact on overall macroeconomic performance and living standards in these countries (Deaton 1999; Cashin et al. 2002; Rossen, 2014).

Precious metals are considered as leading indicators of inflation or as a variable which can transmit the outlook of monetary policy to the economy (Greenspan, 1993). In other words, the pro-cyclical character of the demand for precious metals has underlined their roles as safe-havens against inflation and stores of value and may provide important information as to where the real economy is heading (Baur and Lucey, 2010; Gil-Alana et al., 2015). Thus, precious metals offer valuable diversification opportunities to investors and serve as monetary medium when the market is uncertain (Arouri et al., 2012; Batten et al., 2010).

Due to their diverse industrial uses and investment purposes, metals play an important role in the construction industry, the electrical industry and the automotive manufacturing among others (Rossen, 2014). Gold for instance is used in various sectors (e.g., jewellery, electronics, computers, medical, and aerospace) and is viewed as the most influential metal because of its multifunctions such as storage of value, reserve for money issuance, anti-inflation shelter and financial instrument among others (Mui and Chu, 1993). The gold market operates as a type of insurance against extreme movements in the value of traditional assets during unstable financial markets (Shafiee and Topal, 2010). It takes the form of a store of value held in the inventories of central banks as a reserve that keeps its purchasing power and acts as a stabilizing force in the financial system, reducing losses in the face of negative swings in the market (Baur and Mcdermott, 2010). Silver also has multiple industrial and investment uses which has the potential of making its price more volatile than other commodities (Harper et al., 2013). Silver can be transformed from its natural state and used in the technology and medical industries to produce items such as solar energy, water purification, and X-Ray devices. Moreover, silver is also used in the electronics, and automobile industries to produce components for computers and antifreeze materials. Further, silver can also be used as an investment vehicle by investors who seek profits or to diversify their investment portfolio or hedge (Harvey et al., 2012).

Overall, understanding the price developments in the gold and silver markets may help in forecasting their prices as well as aiding investment decisions and economic planning. This also has important implications for profitable capacity expansion by both private and government-owned mining companies (Davis and Samis, 2006; Radetzki 2008; Arezki et al. 2014).

A number of studies have modelled the trend and cycle features of commodity prices using methods which range from informal graphical inspection of the data to rigorous statistical decomposition techniques and recently to fractional integration approach. The results from these studies are mixed. For instance, Cheung and Lai (1993) used the rescaled range technique and showed that gold returns exhibit long memory but the authors showed that this is mostly due to a small number of observations relating to Middle Eastern political tension and the activities of the Hunt Brothers in 1979. Arouri et al (2012) used several parametric and semiparametric methods including ARFIMA- FIGARCH model and found strong evidence of long range dependence in the daily conditional return and volatility processes of four precious metals: gold, silver, platinum and palladium. Uludag and Lkhamazhapov (2014) used a similar approach as Arouri et al (2012), and found evidence of anti-persistence in spot returns and a lack of long memory property in gold futures returns. They concluded that long memory is a true feature of the data and not due to structural breaks. Ewing and Malik (2013) used the GARCH model and found evidence of persistence in the volatility of gold with and without structural breaks. Gil-Alana et al. (2015) analysed the persistence properties of five metal prices- gold, silver, platinum, palladium, and rhodium- within a fractional integration framework while accounting for structural breaks. In general they find evidence of long memory behaviour and hence long range dependence across time in the precious metals examined.

Laby's et al. (2000) examine the chronology, frequency, duration, and amplitude of twenty one primary commodity price series (including gold and silver) over the period 1960 to 1995. Using Structural Time Series (STS) method, they provide evidence for cyclical behavior in the expansion, contraction, and overall phases for a number of commodities. Their results suggest the predominance of two kinds of cycles at least at that stage. Over the total time span the first cycle usually shows a periodicity of less than twelve months or one year. This was confirmed for most of the commodities. The second cycle shows a periodicity close to two years or more. This is particularly evident for cocoa, copper, corn and tea, though the amplitude of the cycles is low. Ciner (2001) examined the long run trend in prices of gold and silver futures contracts listed on the Tokyo Commodity Exchange. Using daily closing prices from 1992 to 1998 along with Johansen's cointegration analysis, the results indicated that the long run stable relationship between gold and silver future prices had disappeared.

Cashin and McDermott (2002) employ the industrial commodity price index of The Economist, covering the period 1862-1999 and find a downward trend of 1.3% per annum in real commodity prices with no breaks in the trend function. Zanas (2005) employs index of 24 annual non-fuel primary commodity prices from 1900 to 1998 and identifies two structural breaks (1920 and 1984) and shows the 20th century decline in commodity prices is almost exclusively explained by these shifts in the mean as opposed to any gradual trending effect.

Cuddington and Jerrett (2008) use band-pass (BP) filters developed by Baxter and King (1999) and Christiano and Fitzgerald (2003) to search for evidence of super cycles in price of six base metals traded in London Metal Exchange (LME) - aluminum, copper, lead, nickel, tin, and zinc. They find considerable evidence of three past super cycles in real metal prices, defined as cyclical components with expansion phases from 10 to 35 years. They also show that the amplitude of the super cycles is large with variations of 20 to 40% above and below the long-run trends. Harvey et al. (2010) employ a unique disaggregated dataset,

comprised of 25 separate commodity time series and spanning the 17th to the 21st centuries. Using test procedures for the presence of a linear trend and a broken trend which are robust to whether shocks are generated by an  $I(0)$  or  $I(1)$  process, they show that five price series (Aluminum, Banana, Rice, Sugar and Tea) present a significant and downward trend over all or some fraction of the sample period. Of the remainder, nineteen reveal a zero trend (Beef, Coal, Cocoa, Coffee, Copper, Cotton, Gold, Hide, Jute, Lamb, Lead, Nickel, Oil, Pig Iron, Silver, Tin, Wheat, Wool and Zinc), whilst only Tobacco has a significant positive trend over the full sample period. Jacks et al. (2011), examine eleven price datasets that overlap intertemporally from 1700- 2005. Their results suggest that nominal commodity prices have typically provided greater volatility than manufactures, even prior to the industrial revolution, noting that volatility itself is not a function of time per se, but increase in periods of war or autarky and decrease in periods of internationalisation and trade.

Harvey et al. (2012) disentangle trend and cyclical components for relative commodity prices using new and ultra-long aggregate commodity price time series, covering the period 1650- 2010. Employing quasi-feasible GLS-based testing approach, it is shown that the trend path of the commodity series can be split into four regimes (i.e. 1650 to the early 1820s, the early 1820s to the early 1870s, the early 1870s to the mid-1940s, and the mid-1940s to 2010). Moreover, using Christiano and Fitzgerald (2003) BP filter, they find that long-run cycles (often called super-cycles) last for approximately twenty seven years, but appear to be increasing in periodicity over the 20th century, whereas shorter-run cycles last for approximately four years but appear to be decreasing in periodicity.

Gutiérrez et al. (2013) use the Dow Jones Industrial Average and Gold Fix ratio to establish the evolution of gold prices in relation to the NYSE. Results based on Fourier analysis shows that gold prices are found to evolve cyclically with a dominant period of 37 years. Rossen (2014) explores the dynamics (co-movement, price cycles and long-run trends)

of monthly twenty metal prices including gold and silver during January 1910 and December 2011. Results based on the asymmetric band-pass (BP) filter by Christiano and Fitzgerald (2003) show that price cycles are asymmetric; the number of cycles varies significantly depending on the specific metal under consideration, the majority of the metal price series can be characterized by four super cycles consistent with previous studies. However, there is no evidence of duration dependence and the long-run component of metal prices considerably varies over the set of mineral commodities.

In this paper we propose an alternative modelling specification for the real prices of gold and silver. Classical methods include the trend stationary  $I(0)$  and the nonstationary unit roots ( $I(1)$ ) models. However, during the last twenty years fractional integration has become an alternative plausible way of modelling many economic time series. This specification imposes the existence of a pole or singularity in the spectrum at the zero frequency, which is usually associated with the long run trend behaviour. However, another inherent feature observed in many series, including gold and silver prices, is the existence of a cyclical pattern that many times is reduced to the short run dynamics and incorporated throughout a simple  $AR(2)$  process with complex roots. In this paper, we incorporate cycles in the long run dynamics by means of allowing the existence of another pole in the spectrum at a non-zero frequency. Thus, we consider three different specifications: a) a standard  $I(d)$  process, with a pole or singularity at the zero frequency; b) a cyclical  $I(d)$  model that incorporates a single pole in the spectrum at a non-zero frequency, and c) a general model that incorporates the two features in a single framework and that includes two poles in the spectrum of the series. Therefore, this latter model incorporates two fractional differencing parameters, one at the long run or zero frequency and another one at a non-zero (cyclical) frequency. To the best of our knowledge, this is the first attempt to model trend and cyclical properties of long-span gold and silver prices simultaneously in a fractional integration framework.



The outline of the paper is as follows: Section 2 describes the models employed in the paper and the methodology used. Section 3 presents the data and the main empirical results, while Section 4 concludes the paper.

## 2. The models

Assuming that  $\{u_t, t = 0, \pm 1, \dots\}$  is a covariance stationary  $I(0)$  process, three models are examined in this work.

### a) The standard $I(d)$ model

It is specified as:

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, \dots, \quad (1)$$

where  $d$  can be any real value,  $L$  is the lag-operator ( $Lx_t = x_{t-1}$ ) and  $u_t$  is  $I(0)$ , defined for our purposes as a covariance stationary process with a spectral density function that is positive and finite at any frequency in the spectrum.

The  $I(d)$  model of the form given by equation (1) were introduced by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981) and since then have been widely employed to describe the behaviour of many economic time series (see, e.g., Diebold and Rudebusch, 1989; Sowell, 1992a; Gil-Alana and Robinson, 1997; etc.). Note that the parameter  $d$  plays a crucial role in describing the degree of dependence (long run persistence) of the series. Specifically, if  $d = 0$  in (1),  $x_t = u_t$ , and the series is  $I(0)$ , potentially including ARMA structures with the autocorrelations decaying at an exponential rate. If  $d$  belongs to the interval  $(0, 0.5)$ , the series is still covariance stationary but the autocorrelations take longer to disappear than in the  $I(0)$  case. If  $d$  is in the interval  $[0.5, 1)$ , the series is no longer covariance stationary; however, it is still mean-reverting with shocks affecting it disappearing in the long run. Finally, if  $d \geq 1$  the series is nonstationary and non-mean-reverting.

There exist many methods for estimating and testing  $d$  in equation (1). Some of them are parametric while others are semiparametric and they can be specified in the time or in the frequency domain. In this paper we use a parametric method that uses the Whittle function in the frequency domain (Dahlhaus, 1989) along with a Lagrange Multiplier (LM) test proposed by Robinson (1994) and that it has been used in numerous empirical applications (Gil-Alana and Robinson, 1997; Gil-Alana, 2000; Gil-Alana and Henry, 2003; etc.) .

**b) The cyclical I(d) model**

This specification is based on the Gegenbauer process, and it is defined as:

$$(1 - 2 \cos w_r L + L^2)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (2)$$

where  $w_r$  and  $d$  are real values, and  $u_t$  is  $I(0)$ . For practical purposes we define  $w_r = 2\pi r/T$ , with  $r = T/s$ , and thus  $s$  will indicate the number of time periods per cycle, while  $r$  refers to the frequency that has a pole or singularity in the spectrum of  $x_t$ . Note that if  $r = 0$  (or  $s = 1$ ), the fractional polynomial in (2) becomes  $(1 - L)^{2d}$ , which is the polynomial associated with the previous case of fractional integration at the long-run or zero frequency. This type of process was introduced by Andel (1986) and subsequently analysed by Gray, Zhang and Woodward (1989, 1994), Giraitis and Leipus (1995), Chung (1996a,b), Gil-Alana (2001a) and Dalla and Hidalgo (2005) among others. It can be shown that denoting  $\mu = \cos w_r$ , for all  $d \neq 0$ ,

$$(1 - 2\mu L + L^2)^{-d} = \sum_{j=0}^{\infty} C_{j,d}(\mu) L^j,$$

where  $C_{j,d}(\mu)$  are orthogonal Gegenbauer polynomial coefficients recursively defined as:

$$C_{0,d}(\mu) = 1, \quad C_{1,d}(\mu) = 2\mu d,$$

$$C_{j,d}(\mu) = 2\mu \left( \frac{d-1}{j} + 1 \right) C_{j-1,d}(\mu) - \left( 2 \frac{d-1}{j} + 1 \right) C_{j-2,d}(\mu), \quad j = 2, 3, \dots,$$

(see, for instance, Magnus et al., 1966, Rainville, 1960, etc. for further details on Gegenbauer polynomials). Gray et al. (1989) showed that  $x_t$  in (2) is (covariance) stationary if  $d < 0.5$  for  $|\mu = \cos w_r| < 1$  and if  $d < 0.25$  for  $|\mu| = 1$ .

Here, we will employ first a testing parametric method of Dalla and Hidalgo (2005). It tests the null hypothesis of no cycles against the alternative of strong cycles of form as in (2). Additionally, we will employ the methods of Giraitis et al. (2001) and Hidalgo (2005), testing both  $r$  and  $d$  in equation (2) in a parametric and a semiparametric way respectively.

**c) A model with two poles at the spectrum**

The third model combines the two previous approaches in a single framework. Thus, the model examined in the following:

$$(1 - L)^{d_1} (1 - 2 \cos w_r L + L^2)^{d_2} x_t = u_t, \quad t = 1, 2, \dots, \quad (3)$$

where  $d_1$  is the order of integration corresponding to the long-run or zero frequency, and  $d_2$  is the order of integration with respect to the non-zero (cyclical) frequency, and  $u_t$  is assumed once more to be an  $I(0)$  process. As in the previous cases,  $d_1$  and  $d_2$  are allowed to be real values and thus they are not restricted to be integers.

Dealing with the model in equation (3), Robinson (1994) propose a Lagrange Multiplier (LM) test

$$H_o : d \equiv (d_1, d_2)^T = (d_{1o}, d_{2o})^T \equiv d_o, \quad (4)$$

in (3), where  $x_t$  are the regression errors in a model of form:

$$y_t = \beta^T z_t + x_t, \quad t = 1, 2, \dots, \quad (5)$$

where  $y_t$  is the observed time series;  $\beta$  is a  $(k \times 1)$  vector of unknown parameters, and  $z_t$  is a  $(k \times 1)$  vector of deterministic terms, that might include, for example, an intercept (i.e.  $z_t = 1$ ) or an intercept with a linear trend ( $z_t = (1, t)^T$ ). The specific form of the test statistic (denoted

by  $\hat{R}$ ) is presented in the Appendix. Under very general regularity conditions, Robinson (1994) showed that for this particular version of his tests,

$$\hat{R} \rightarrow_d \chi_2^2, \quad \text{as } T \rightarrow \infty, \quad (6)$$

where  $T$  indicates the sample size, and “ $\rightarrow_d$ ” stands for convergence in distribution. Thus, unlike in other procedures, we are in a classical large-sample testing situation. A test of (5) will reject  $H_0$  against the alternative  $H_a: d \neq d_0$  if  $\hat{R} > \chi_{2,\alpha}^2$ , where  $\text{Prob}(\chi_2^2 > \chi_{2,\alpha}^2) = \alpha$ . There are several reasons for using this approach. First, this test is the most efficient in the Pitman sense against local departures from the null, that is, if it is implemented against local departures of the form:  $H_a: d = d_0 + \delta T^{-1/2}$ , for  $\delta \neq 0$ , the limit distribution is a  $\chi_2^2(\nu)$ , with a non-centrality parameter  $\nu$  that is optimal under Gaussianity of  $u_t$ . Moreover, Gaussianity is not necessary for the implementation of this procedure, a moment condition of only order 2 being required.

### 3. Data and empirical results

The data examined correspond to the annual data for real prices of gold (1833 – 2013) and silver (1792 – 2013) in natural logarithmic form, with the start and end points of the samples purely driven by data availability at the time of writing this paper. The data on nominal prices (London PM Fix US dollar per ounce) for gold and silver is obtained from [www.kitco.com](http://www.kitco.com), while the annual Consumer Price Index (CPI) data used to deflate the nominal prices to obtain the real values of gold and silver prices, are derived from the website of Professor Robert Sahr (<http://oregonstate.edu/cla/polisci/sahr/sahr>). Figure 1 displays the time series plots (in logs) along with their first differences. The first differenced data display some heteroscedastic behaviour but the methods employed in this paper for estimating and testing the differencing parameters seem to be robust to this feature.

**[Insert Figures 1 – 3 about here]**

The correlograms and the periodograms of the series and the first differenced data are displayed in Figures 2 and 3 respectively. The correlograms show a cyclical pattern that is clearer observed for the gold real prices. The periodograms of the original data show the highest value at the zero frequency while those of the first differenced data display the highest values at a non-zero frequency.

We start by presenting the results for the case of the standard case of I(d) models. Table 1 displays the estimates of d (and the 95% confidence interval) for the three standard cases of no regressors, an intercept, and an intercept with a linear time trend, assuming that  $u_t$  is a white noise process, an AR(1) and that it follows the model of Bloomfield, which is a non-parametric approach of modelling the I(0) error term.<sup>1</sup>

**[Insert Table 1 about here]**

Starting with the deterministic terms, the first thing we note is that an intercept seems to be sufficient to describe the deterministic part of the model. Focussing first on the gold prices, we see that if  $u_t$  is white noise, the estimated value of d is 1.32, and the unit root null hypothesis is rejected in favour of higher orders of integration, however, allowing for autocorrelated disturbances, the estimated ds are below 1, and the unit root is almost rejected. Looking at the silver log-prices, the three estimates of d are below 1, the unit root cannot be rejected with white noise errors, and this hypothesis is decisively rejected in favour of mean reversion ( $d < 1$ ) with autocorrelated errors.

Next we focus on strong cycles and consider the model given by equation (2). Here first we perform the Dalla and Hidalgo's (2005) method and the results reject the null hypothesis of no cycles in the two series in favour of cyclical long range dependence. Estimating the location for the pole or singularity at the spectrum with the semiparametric method of Hidalgo (2005), the estimates range in the two series between 4 and 9 periods

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<sup>1</sup> This nonparametric approach of Bloomfield (1973) accommodates very well in the context of fractional integration (Velasco and Robinson, 2000; Gil-Alana, 2001b, 2004; etc.)

depending on the choice of the bandwidth parameter number. Tables 2a and 2b displays the estimates of  $d$  in (2) for  $r = 4, 5, 6, 7$  and  $8$ , again for the three cases of no regressors, an intercept, and an intercept with a linear trend. Table 2a refers to the case of white noise disturbances, while Table 2b focuses on autocorrelated AR(1) errors.

**[Insert Tables 2a and 2b about here]**

As with the previous model, the most significant results are those based on an intercept but with no linear trend. Generally the estimates of  $d$  are positive in the two series though more evidence of significant positive values of  $d$  is found in the gold real prices data. Employing the exponential spectral model of Bloomfield (1973) for the  $I(0)$  error term  $u_t$  in (2) produced estimates of  $d$  fairly similar to those reported in Table 2b for the case of AR(1) errors.

Finally, we employ model (3), which is a more flexible specification than (1) and (2) in the sense that it allows us to jointly estimated the fractional differencing parameters corresponding to the zero and non-zero frequencies. Table 3a refers to the case of white noise errors, while Tables 3b and 3c displays the estimates for the cases of AR(1) and Bloomfield-type disturbances. Focussing on the case with an intercept and starting with Table 3a (white noise) we see that for gold,  $d_1$  is above 1 in all cases ranging from 1.15 ( $r = 7, 8$ ) to 1.30 ( $r = 5$ ), while  $d_2$  is slightly above 0.2 for 4 values of  $r$  and close to 0 if  $r = 4$ . Looking at the results for silver in the same table,  $d_1$  is slightly below 1 and  $d_2$  is slightly above 0 for the five values presented for  $r$ .

**[Insert Tables 3a, 3b and 3c about here]**

Table 3b refers to AR(1) disturbances. For gold, the estimates of  $d$  ranges between 0.53 ( $r = 5$ ) and 0.89 ( $r = 6, 7$ ), while  $d_2$  is equal to 0.03 with  $r = 4$  and it ranges between 0.12 and 0.17 in the remaining cases. For silver,  $d_1$  oscillates between 0.55 ( $r = 8$ ) and 0.68 ( $r = 4$ ), while  $d_2$  is very close to 0. Using the exponential spectral model of Bloomfield (1973) (in

Table 3c) the estimates are very close to those based on white noise disturbances. Thus, for gold  $d_1$  is above 1 and  $d_2$  above 0, and for silver, the estimates of  $d_1$  are slightly below 1 while those of  $d_2$  are slightly above 0. Thus, we can conclude by saying that higher orders of integration are associated to the long run component compared with the cyclical one. Thus, the estimates associated to the long run or zero frequency seem to be above 1 in case of gold and below 1 for silver, while the order of integration associated with the cyclical frequency is slightly above 0 in the majority of the cases in the two series.

#### **4. Conclusions**

The study proposes an alternative modelling specification for the real prices of gold and silver. We use annual data covering the period 1883-2013. We consider both the long run trend behaviour and the cycle patterns in these two series within a fractional integration framework. We incorporate cycles in the long run dynamics by means of allowing the existence of additional pole in the spectrum at a non-zero frequency. In other words, the model incorporates two fractional differencing parameters, one at the long run or zero frequency and another one at a non-zero (cyclical) frequency. This is an extension to the conventional fractional integration model that imposes the existence of a pole or singularity in the spectrum at the zero frequency, usually associated with the long run trend behaviour. However, we also consider the separate cases of a standard  $I(d)$  process, with a pole or singularity at the zero frequency and a cyclical  $I(d)$  model that incorporates a single pole in the spectrum at a non-zero frequency. We consider the estimates of the differencing parameter for the three standard cases of no regressors, an intercept, and an intercept with a linear time trend, and assuming that the residual is a white noise process, an  $AR(1)$  and that it follows the model of Bloomfield. The most significant specification is that with intercept only.

Results based on the standard  $I(d)$  process shows that real gold price is non-mean reverting while real silver price is mean reverting. In other words we find evidence of long memory behaviour in the gold price inflation rate but not in the silver inflation rate. Based on the cyclical  $I(d)$  model, the null hypothesis of no cycles in the two series is rejected in favour of cyclical long range dependence and the location for the pole or singularity at the spectrum in the two series is estimated to be between 4 and 9 periods depending on the number of the bandwidth parameter chosen. Finally, results from the more flexible model indicates that in general the estimates associated to the long run or zero frequency appear to be above 1 in case of gold and below 1 for silver, while the order of integration associated with the cyclical frequency is slightly above 0 in the majority of the cases in the two series. Further, higher orders of integration are associated to the long run component compared with the cyclical one.

These results have important implications for policy. For instance in the event of exogenous shocks, the effects will be permanent in gold and strong policy measures need to be adopted to ensure that gold price returns to its original trend. The persistent property in gold and the cyclical long range dependence property in both gold and silver is important for monetary policy especially with respect to inflation targeting since these properties are likely to affect the persistence nature of the aggregate inflation of an economy. This will have implications for economic variables such as interest rates, consumption, investment, and output growth. More so, the findings have implications for portfolio diversification, forecasting of gold and silver prices and overall economic planning.

At this stage, it is important to point out that, since we use long span data, gold and silver prices are likely to have witnessed structural breaks. And as indicated by Cheung and Lai (1993), and Diebold and Inoue (2001), presence of structural breaks can lead to spurious evidence in favour of long-memory. In light of this, future research should be aimed at re-estimating these models by allowing for breaks in the data generating process. In addition, the



forecasting properties of the models developed in this paper, should also be an interesting area of further investigation.

## Appendix

The test statistic proposed by Robinson (1994) for testing  $H_0$  (4) in the model given by equations (3) and (5) is given by:

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a},$$

where  $T$  is the sample size, and

$$\hat{a} = \frac{-2\pi}{T} \sum_j^* \psi(\lambda_j) \mathbf{g}_u(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} \mathbf{g}_u(\lambda_j; \hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{T} \left( \sum_j^* \psi(\lambda_j) \psi(\lambda_j)' - \sum_j^* \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \left( \sum_j^* \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \sum_j^* \hat{\varepsilon}(\lambda_j) \psi(\lambda_j)' \right)$$

$$\psi(\lambda_j)' = [\psi_1(\lambda_j), \psi_2(\lambda_j)]; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log \mathbf{g}_u(\lambda_j; \hat{\tau}); \quad \psi_1(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|;$$

$\psi_2(\lambda_j) = \log \left| 2(\cos \lambda_j - \cos w_r) \right|$ , with  $\lambda_j = 2\pi j/T$ , and the summation in  $*$  is over all frequencies which are bounded in the spectrum.  $I(\lambda_j)$  is the periodogram of

$$\hat{u}_t = (1-L)^{d_{1o}} (1 - 2\cos w_r L + L^2)^{d_{2o}} y_t - \hat{\beta}' \bar{z}_t, \text{ with}$$

$$\hat{\beta} = \left( \sum_{t=1}^T \bar{z}_t \bar{z}_t' \right)^{-1} \sum_{t=1}^T \bar{z}_t (1-L)^{d_{1o}} (1 - 2\cos w_r L + L^2)^{d_{2o}} y_t;$$

$$\bar{z}_t = (1-L)^{d_{1o}} (1 - 2\cos w_r L + L^2)^{d_{2o}} z_t, \text{ evaluated at } \lambda_j = 2\pi j/T \text{ and } \hat{\tau} =$$

$\arg \min_{\tau \in T^*} \sigma^2(\tau)$ , with  $T^*$  as a suitable subset of the  $\mathbb{R}^q$  Euclidean space. Finally, the

function  $\mathbf{g}_u$  above is a known function coming from the spectral density of  $u_t$ :

$$f(\lambda) = \frac{\sigma^2}{2\pi} \mathbf{g}_u(\lambda; \tau), \quad -\pi < \lambda \leq \pi.$$

Note that these tests are purely parametric and, therefore, they require specific modelling assumptions about the short-memory specification of  $u_t$ . Thus, if  $u_t$  is white noise,  $\mathbf{g}_u \equiv 1$ , and

if  $u_t$  is an AR process of the form  $\phi(L)u_t = \varepsilon_t$ ,  $g_u = |\phi(e^{i\lambda})|^{-2}$ , with  $\sigma^2 = V(\varepsilon_t)$ , so that the AR coefficients are a function of  $\tau$ .

The point estimates were obtained by choosing the values that minimise Robinson's (1994) test statistic over a grid of values for  $d_1$ ,  $d_2$  and  $r$ . These parameter estimates were practically identical to those obtained by maximising the Whittle function in the frequency domain (Dahlhaus, 1989). The confidence intervals were obtained by choosing the values of the differencing parameters were the null hypotheses tested could not be rejected at the 5% level.

## References

- Agiakloglou, C. and P. Newbold (1994) Lagrange multiplier tests for fractional difference. *Journal of Time Series Analysis*, 14, 253–262.
- Andel, J. (1986) Long memory time series models. *Kybernetika* 22, 105–123.
- Arezki, R., Loungani, P., van der Ploeg, R. and Venables, A. (2014) Understanding international commodity price fluctuations. *Journal of International Money and Finance*, 42 (C), 1–8.
- Arouri, M., Hammoudeh, S., Lahiani, A. and Nguyen, D. (2012) Long memory and structural breaks in modelling the return and volatility dynamics of precious metals. *The Quarterly Review of Economics and Finance*, 52, 207–218.
- Batten, J. A., Ciner, C. and Lucey, B. M. (2010) The macroeconomic determinants of volatility in precious metals markets. *Resources Policy*, 35, 65–71.
- Baur, D. G. and Lucey, B. M. (2010) Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *Financial Review*, 45(2), 217–229.
- Baur, D.G. and McDermott, T. K. (2010) Is gold a safe haven? International evidence. *Journal of Banking and Finance*, 34(8), 1886–1898.
- Baxter, M. and King, R.G. (1999) Measuring business cycles: Approximate band-pass filters for economic time series. *Review of Economics and Statistics*, 81, 575–593.
- Beran, J. (1995) Maximum likelihood estimation of the differencing parameter for invertible short and long memory autoregressive integrated moving average models. *Journal of the Royal Statistical Society B*, 57, 659–672.
- Blattman, C., Hwang, J. and Williamson, J.G. (2007) Winners and losers in the commodity lottery: The impact of terms of trade growth and volatility in the periphery 1870-1939. *Journal of Development Economics*, 82, 156–179.
- Cashin, P., McDermott, C. and Scott, A. (2002) Booms and slumps in world commodity prices. *Journal of Development Economics*, 69 (1), 277–296.
- Cashin, P. and McDermott, C.J. (2002) The Long-run behaviour of commodity prices: small trends and big variability. *IMF Staff Papers*, 49, 175–199.
- Cheung, Y-W and Lai, K.S. (1993) Do gold market returns have long memory? *Financial Review*, 28 (2), 181–202.
- Christiano, L. and Fitzgerald, T.J. (2003) The Bandpass filter. *International Economic Review*, 44, 435–465.
- Chung, C.-F. (1996a) A generalized fractionally integrated autoregressive moving-average process. *Journal of Time Series Analysis*, 17, 111–140.

- Chung, C.-F.(1996b) Estimating a generalized long memory process. *Journal of Econometrics*, 73, 237–259.
- Ciner, C. (2001). On the relationship between gold and silver prices: A note. *Global Finance Journal*, 12, 299–303.
- Cuddington, J.T. and Jerrett, D. (2008) Super cycles in real metals prices? *IMF Staff Papers* 55 (4), 541–565.
- Dahlhaus, R. (1989) Efficient parameter estimation for self-similar process. *Annals of Statistics*, 17, 1749–1766.
- Dalla, V. and Hidalgo, J. (2005) A parametric bootstrap test for cycles. *Journal of Econometrics*, 129, 219–261.
- Davis, G., and Samis, M. (2006) Using real options to manage and value exploration. *Society of Economic Geologists Special Publication*, 12(14), 273–94.
- Deaton, A. (1999) Commodity prices and growth in Africa. *Journal of Economic Perspectives*, 13 (3), 23–40.
- Diebold, F.X., and Inoue, A. (2001). Long memory and regime switching. *Journal of Econometrics*, 105, 131–159.
- Diebold, F.X. and Rudebusch, G. D. (1989) Long memory and persistence in aggregate output. *Journal of Monetary Economics*, 24, 189–209.
- Erten, B. and Ocampo, J. A. (2013) Super cycles of commodity prices since the Mid-nineteenth Century. *World Development*, 44 (C), 14–30.
- Ewing, B. and Malik, F. (2013) Volatility transmission between gold and oil futures under structural breaks. *International Review of Economics and Finance*, 25, 113–121.
- Geweke, J. and Porter-Hudak, S. (1983) The estimation and application of long memory time series models. *Journal of Time Series Analysis*, 4(4), 221–238.
- Gil-Alana, L. A. (2000) Mean reversion in the real exchange rates. *Economics Letters*, 16, 285–288.
- Gil-Alana, L.A. (2001a) Testing stochastic cycles in macroeconomic time series. *Journal of Time Series Analysis*, 22, 4, 411–430.
- Gil-Alana, L.A. (2001b) An exponential spectral model for the UK unemployment. *Journal of Forecasting*, 20, 329–340.
- Gil-Alana, L. A. (2004). The use of the Bloomfield (1973) model as an approximation to ARMA processes in the context of fractional integration. *Mathematical and Computer Modelling*, 39, 429–436.

- Gil-Alana, L.A. and Henry, B. (2003) Fractional integration and the dynamics of UK unemployment. *Oxford Bulletin of Economics and Statistics*, 65(2), 221–239.
- Gil-Alana, L.A. and Robinson, P.M. (1997) Testing of unit roots and other nonstationary hypotheses in macroeconomic time series. *Journal of Econometrics*, 80, 241–268.
- Gil-Alana, L.A., Chang, S. Balcilar, M., Aye, G.C. and Gupta, R. (2015) Persistence of precious metal prices: A fractional integration approach with structural breaks. *Resources Policy*, 44, 57–64.
- Giriatis, L., J. Hidalgo and P.M. Robinson (2001) Gaussian estimation of parametric spectral density with unknown pole. *Annals of Statistics* 29, 987–1023.
- Giraitis, L. and Leipus, R. (1995) A generalized fractionally differencing approach in long memory modeling. *Lithuanian Mathematical Journal*, 35, 65–81.
- Granger, C.W.J. (1980) Long memory relationships and the aggregation of dynamic models. *Journal of Econometrics*, 14, 227–238.
- Granger, C.W.J. (1981) Some properties of time series data and their use in econometric model specification. *Journal of Econometrics*, 16, 121–130.
- Granger, C.W.J. and Joyeux, R. (1980) An introduction to long memory time series and fractionally differencing. *Journal of Time Series Analysis*, 1, 15–29.
- Gray, H.L., Yhang, N. and Woodward, W.A. (1989) On generalized fractional processes. *Journal of Time Series Analysis*, 10, 233–257.
- Gray, H.L., Yhang, N., Woodward, W.A. (1994) On generalized fractional processes. A correction. *Journal of Time Series Analysis*, 15, 561–562.
- Greenspan, A. (1993) The Fed aims for price stability. *Challenge*, 4–10.
- Gutiérrez, M., Franco, G. and Campuzano, C. (2013) Gold prices: Analyzing its cyclical behaviour. *Lect. Econ. no.79 Medellín July/Dec. 2013*.  
<http://www.scielo.org.co/scielo.php?pid=S0120-25962013000200004&s...>
- Harper, A., Jin, Z., Sokunle, R., and Wadhwa, M. (2013) Price volatility in the silver spot market: An empirical study using GARCH applications. *Journal of Finance and Accountancy*, 13(1), 1-11.
- Harvey, D.I., Kellard, N.M., Madsen, J.B. and Wohar, M.E. (2012) Trends and cycles in real commodity prices: 1650-2010. Centre for Economic History, the Australian National University Discussion Paper No. 2012-10.
- Harvey, D.I., Kellard, N.M., Madsen, J.B. and Wohar, M.E. (2010) The Prebisch-Singer hypothesis: Four centuries of evidence. *Review of Economics and Statistics*, 92, 367–377.
- Hidalgo, J. (2005). Semiparametric estimation for stationary processes whose spectra have an unknown pole. *Annals of Statistics*, 33(4), 1843–1889.

- Hosking, J.R.M. (1981) Fractional differencing. *Biometrika*, 68, 168–176.
- Jacks, D.S., O'Rourke, K.H. and Williamson, J.G. (2011) Commodity price volatility and world market integration. *Review of Economics and Statistics*, 93, 800–813.
- Labys, W. (2006) Modeling and forecasting primary commodity prices. Chippenham, UK: Ashgate Publishing.
- Labys, W.C., Kouassi, E. and Terraza, M. (2000) Short-term cycles in primary commodity prices. *The Developing Economies*, XXXVIII-3, 330–42
- Magnus, W., Oberhettinger, F. and Soni, R.P. (1966) Formulas and theorem for the special functions of mathematical physics. Springer, Berlin.
- Mui, H. W. and Chu, C. W. (1993) Forecasting the spot price of gold: combined forecast approaches versus a composite forecast approach. *Journal of Applied Statistics*, 20(1), 13–23.
- Poelhekke, S. and van der Ploeg, R. (2009) Volatility and the natural resource curse. *Oxford Economic Papers*, 61, 727–760.
- Radetzki, M. (2008) A handbook of primary commodities in the global economy Cambridge, Cambridge University Press.
- Rainville, E.D. (1960) Special functions. MacMillan, New York.
- Robinson, P.M. (1994) Efficient tests of nonstationary hypotheses. *Journal of the American Statistical Association*, 89, 1420–1437.
- Robinson, P.M. (1995a). Gaussian semiparametric estimation of long range dependence. *Annals of Statistics*, 23, 1630–1661.
- Robinson, P. M. (1995b) Log-periodogram regression of time series with long range dependence. *Annals of Statistics*, 23, 1048–1072.
- Rossen, A. (2014). What are metal prices like? Co-movement, price cycles and long-run trends. Hamburg Institute of International Economics (HWWI), Research Paper No. 155.
- Shafiee, S. and Topal, E. (2010) An overview of global gold market and gold price forecasting. *Resources Policy*, 35, 178–189.
- Sowell, F. (1992a) Modelling long run behaviour with the fractional ARIMA model. *Journal of Monetary Economics*, 29, 277–302.
- Sowell, F.B. (1992b) Maximum likelihood estimation of stationary univariate fractionally integrated time series models. *Journal of Econometrics*, 53, 165–188.
- Uludag, B.K. and Lkhamazhapov, Z. (2014) Long memory and structural breaks in the returns and volatility of Gold: evidence from Turkey. *Applied Economics*, 46(31), 3777–3787.

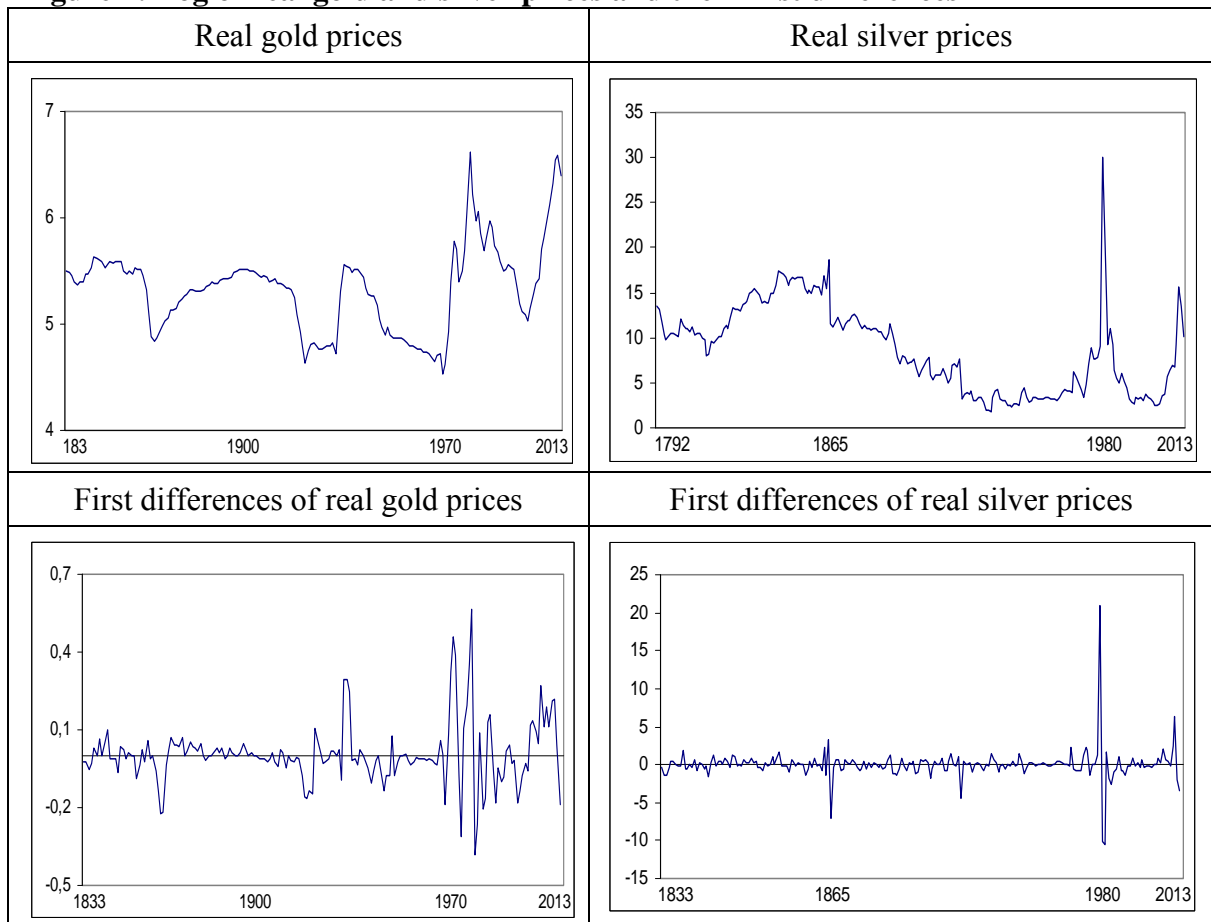
Velasco C. and Robinson, P.M. (2000) Whittle pseudo-maximum likelihood estimation for nonstationary time series. *Journal of the American Statistical Association*, 95, 1229–1243.

Zanias, G. (2005). Testing for trends in the terms of trade between primary commodities and manufactured goods. *Journal of Development Economics*, 78, 49–59.

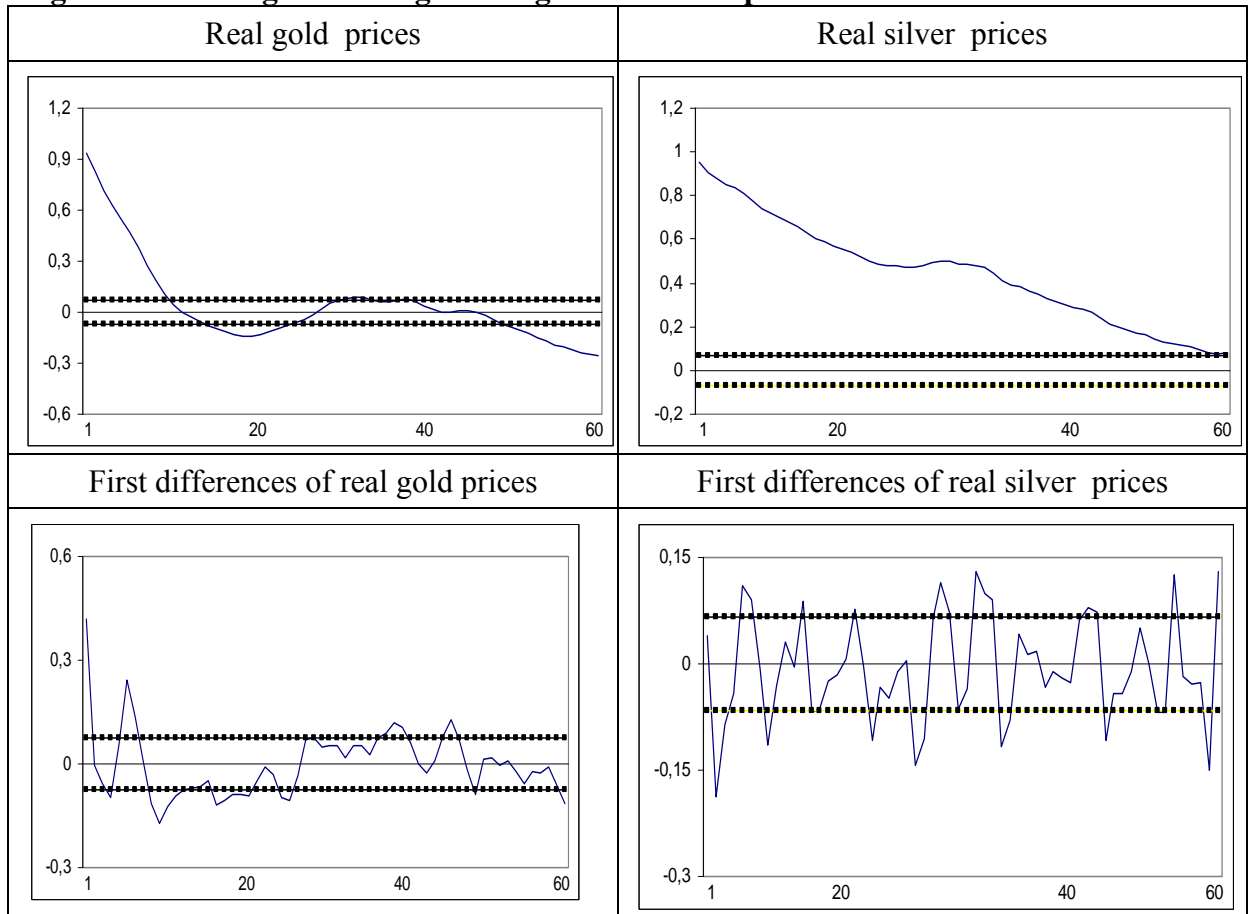


## TABLES AND FIGURES

**Figure 1: Log of real gold and silver prices and their first differences**

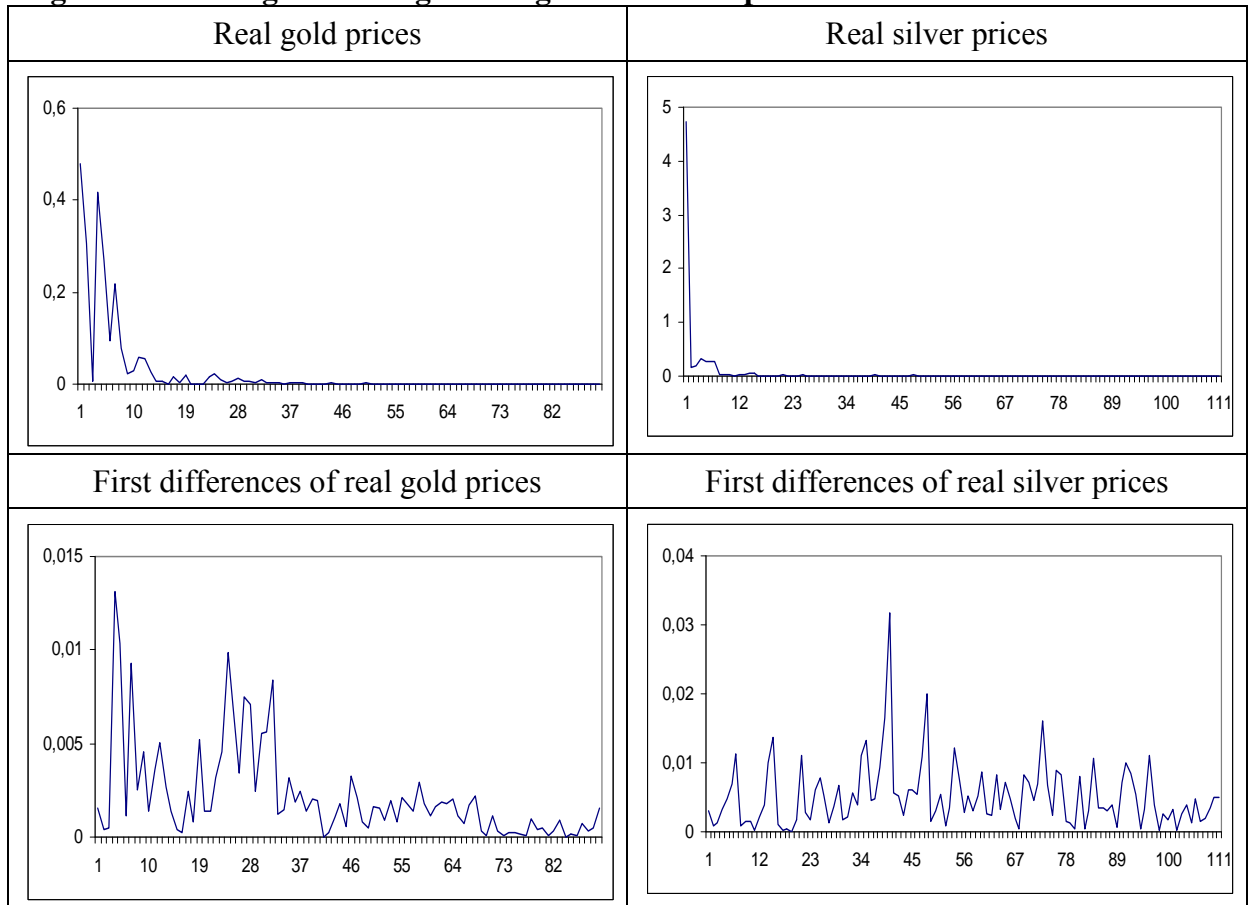


**Figure 2: Correlograms of log of real gold and silver prices and their first differences**



The thick lines give the 95% confidence band for the null hypothesis of no autocorrelation.

**Figure 3: Periodograms of log of real gold and silver prices and their first differences**



The horizontal axis refers to the discrete Fourier frequencies  $\lambda_j = 2\pi j/T$ ,  $j = 1, \dots, T/2$ .

**Table 1: Estimates of d in an I(d) model at the zero frequency (Model (1))**

i) Log of real gold prices			
$u_t$ (disturbances)	No regressors	An intercept	A linear time trend
White noise	1.00 (0.92, 1.10)	<b>1.32 (1.17, 1.56)</b>	1.33 (1.17, 1.56)
AR (1)	1.29 (1.13, 1.50)	<b>0.78 (0.57, 0.99)</b>	0.42 (0.16, 0.91)
Bloomfield (1)	1.01 (0.86, 1.21)	<b>0.77 (0.59, 1.00)</b>	0.76 (0.59, 1.00)
ii) Log of real silver prices			
$u_t$ (disturbances)	No regressors	An intercept	A linear time trend
White noise	0.94 (0.87, 1.04)	<b>0.92 (0.82, 1.05)</b>	0.92 (0.82, 1.05)
AR (1)	xxx	<b>0.71 (0.56, 0.88)</b>	0.70 (0.49, 0.88)
Bloomfield (1)	0.93 (0.81, 1.07)	<b>0.72 (0.60, 0.88)</b>	0.71 (0.57, 0.88)

In bold, the significant models according to the deterministic terms.

**Table 2a: Estimates of d in a cyclical I(d) model (Model (2))**

ii) Log of real gold prices			
White noise $u_t$	No regressors	An intercept	A linear time trend
$r = 4$	-0.098 (-0.198, 0.040)	<b>-0.097 (-0.197, 0.041)</b>	-0.092 (-0.193, 0.050)
$r = 5$	0.233 (0.063, 0.460)	<b>0.233 (0.063, 0.460)</b>	0.239 (0.069, 0.466)
$r = 6$	0.318 (0.195, 0.467)	<b>0.318 (0.195, 0.466)</b>	0.318 (0.196, 0.465)
$r = 7$	0.314 (0.207, 0.438)	<b>0.313 (0.207, 0.438)</b>	0.312 (0.206, 0.435)
$r = 8$	0.296 (0.191, 0.413)	<b>0.296 (0.191, 0.412)</b>	0.293 (0.189, 0.409)
ii) Log of real silver prices			
White noise $u_t$	No regressors	An intercept	A linear time trend
$r = 4$	0.092 (-0.018, 0.243)	<b>0.091 (-0.019, 0.243)</b>	0.093 (-0.018, 0.245)
$r = 5$	0.125 (0.035, 0.233)	<b>0.125 (0.035, 0.233)</b>	0.126 (0.036, 0.234)
$r = 6$	0.099 (0.028, 0.182)	<b>0.099 (0.028, 0.182)</b>	0.099 (0.028, 0.182)
$r = 7$	0.040 (-0.041, 0.131)	<b>0.040 (-0.041, 0.132)</b>	0.040 (-0.041, 0.132)
$r = 8$	0.007 (-0.073, 0.098)	<b>0.007 (-0.074, 0.098)</b>	0.006 (-0.074, 0.097)

In bold, the significant models according to the deterministic terms.

**Table 2b: Estimates of d in a cyclical I(d) model (Model (2))**

i) Log of real gold prices			
AR(1)-type $u_t$	No regressors	An intercept	A linear time trend
$r = 4$	0.065 (-0.067, 0.254)	<b>0.066 (-0.066, 0.256)</b>	0.066 (-0.066, 0.257)
$r = 5$	0.117 (-0.002, 0.274)	<b>0.117 (-0.001, 0.275)</b>	0.120 (0.001, 0.280)
$r = 6$	0.142 (0.048, 0.262)	<b>0.142 (0.049, 0.263)</b>	0.146 (0.053, 0.268)
$r = 7$	0.146 (0.054, 0.271)	<b>0.146 (0.053, 0.270)</b>	0.151 (0.057, 0.277)
$r = 8$	0.111 (0.012, 0.239)	<b>0.110 (0.012, 0.238)</b>	0.115 (0.017, 0.245)
ii) Log of real silver prices			
AR(1)-type $u_t$	No regressors	An intercept	A linear time trend
$r = 4$	0.091 (-0.019, 0.242)	<b>0.091 (-0.018, 0.242)</b>	0.093 (-0.018, 0.244)
$r = 5$	0.155 (0.042, 0.303)	<b>0.155 (0.042, 0.303)</b>	0.158 (0.044, 0.307)
$r = 6$	0.159 (0.057, 0.289)	<b>0.159 (0.057, 0.289)</b>	0.162 (0.059, 0.291)
$r = 7$	0.047 (-0.066, 0.195)	<b>0.048 (-0.065, 0.195)</b>	0.049 (-0.063, 0.197)
$r = 8$	-0.039 (-0.142, 0.088)	<b>-0.039 (-0.142, 0.089)</b>	-0.038 (-0.142, 0.089)

In bold, the significant models according to the deterministic terms.

**Table 3a: Estimates of d1 and d2 in a longrun + cyclical I(d) model (Model (3))**

i) Log of real gold prices						
White noise $u_t$	No regressors		An intercept		A linear time trend	
	$d_1$	$d_2$	$d_1$	$d_2$	$d_1$	$d_2$
$r = 4$	1.04	0.74	<b>1.27</b>	<b>-0.06</b>	1.27	-0.06
$r = 5$	1.02	0.91	<b>1.30</b>	<b>0.23</b>	1.30	0.23
$r = 6$	1.00	0.91	<b>1.21</b>	<b>0.25</b>	1.21	0.25
$r = 7$	0.99	0.90	<b>1.15</b>	<b>0.25</b>	1.15	0.25
$r = 8$	1.00	0.88	<b>1.15</b>	<b>0.22</b>	1.15	0.22
ii) Log of real silver prices						
White noise $u_t$	No regressors		An intercept		A linear time trend	
	$d_1$	$d_2$	$d_1$	$d_2$	$d_1$	$d_2$
$r = 4$	0.93	0.07	<b>0.93</b>	<b>0.06</b>	0.93	0.06
$r = 5$	0.90	0.17	<b>0.90</b>	<b>0.14</b>	0.90	0.14
$r = 6$	0.87	0.16	<b>0.86</b>	<b>0.14</b>	0.86	0.14
$r = 7$	0.85	0.13	<b>0.85</b>	<b>0.10</b>	0.85	0.10
$r = 8$	0.86	0.08	<b>0.84</b>	<b>0.08</b>	0.84	0.08

In bold, the significant models according to the deterministic terms.

**Table 3b: Estimates of d1 and d2 in a longrun + cyclical I(d) model (Model (3))**

ii) Log of real gold prices						
AR-type $u_t$	No regressors		An intercept		A linear time trend	
	$d_1$	$d_2$	$d_1$	$d_2$	$d_1$	$d_2$
$r = 4$	0.86	-0.02	<b>0.72</b>	<b>0.03</b>	0.71	0.03
$r = 5$	1.25	0.05	<b>0.53</b>	<b>0.17</b>	0.52	0.18
$r = 6$	1.25	0.05	<b>0.89</b>	<b>0.15</b>	0.89	0.15
$r = 7$	1.25	0.08	<b>0.89</b>	<b>0.15</b>	0.86	0.14
$r = 8$	0.90	0.19	<b>0.76</b>	<b>0.12</b>	0.76	0.12
ii) Log of real silver prices						
AR.-type $u_t$	No regressors		An intercept		A linear time trend	
	$d_1$	$d_2$	$d_1$	$d_2$	$d_1$	$d_2$
$r = 4$	0.68	0.01	<b>0.68</b>	<b>0.01</b>	0.57	0.01
$r = 5$	0.73	0.06	<b>0.61</b>	<b>0.05</b>	0.61	0.05
$r = 6$	0.72	0.06	<b>0.62</b>	<b>0.05</b>	0.62	0.05
$r = 7$	0.69	-0.02	<b>0.57</b>	<b>-0.02</b>	0.54	-0.02
$r = 8$	0.69	-0.07	<b>0.55</b>	<b>-0.06</b>	0.54	-0.06

In bold, the significant models according to the deterministic terms.

**Table 3c: Estimates of d1 and d2 in a longrun + cyclical I(d) model (Model (3))**

iii) Log of real gold prices						
Bloomfield-type $u_t$	No regressors		An intercept		A linear time trend	
	$d_1$	$d_2$	$d_1$	$d_2$	$d_1$	$d_2$
$r = 4$	1.34	0.05	<b>1.34</b>	<b>0.05</b>	1.34	0.05
$r = 5$	1.29	0.03	<b>1.30</b>	<b>0.03</b>	1.30	0.03
$r = 6$	1.21	0.25	<b>1.21</b>	<b>0.25</b>	1.21	0.25
$r = 7$	1.14	0.25	<b>1.15</b>	<b>0.25</b>	1.15	0.25
$r = 8$	1.11	0.24	<b>1.11</b>	<b>0.24</b>	1.11	0.24
ii) Log of real silver prices						
Bloomfield.-type $u_t$	No regressors		An intercept		A linear time trend	
	$d_1$	$d_2$	$d_1$	$d_2$	$d_1$	$d_2$
$r = 4$	0.95	0.05	<b>0.92</b>	<b>0.05</b>	0.92	0.05
$r = 5$	0.90	0.14	<b>0.90</b>	<b>0.14</b>	0.90	0.14
$r = 6$	0.86	0.14	<b>0.86</b>	<b>0.14</b>	0.86	0.14
$r = 7$	0.85	0.10	<b>0.85</b>	<b>0.10</b>	0.85	0.10
$r = 8$	0.85	0.08	<b>0.84</b>	<b>0.08</b>	0.84	0.08

In bold, the significant models according to the deterministic terms.