University of Pretoria
Department of Economics Working Paper Series

Financial Tail Risks and the Shapes of the Extreme Value Distribution: A Comparison between Conventional and Sharia-Compliant Stock Indexes

John W. Muteba Mwamba
University of Johannesburg
Shawkat Hammoudeh
Drexel University
Rangan Gupta
University of Pretoria
Working Paper: 2014-80
December 2014

Department of Economics
University of Pretoria
0002, Pretoria
South Africa
Tel: +27 12 420 2413

Financial Tail Risks and the Shapes of the Extreme Value Distribution: A Comparison between Conventional and Sharia-Compliant Stock Indexes

John W. Muteba Mwamba[*], Shawkat Hammoudeh[**] and Rangan Gupta[***]

Abstract

This paper makes use of two types of extreme value distributions, namely: the generalised extreme value distribution often referred to as the block of maxima method (BMM), and the peak-over-threshold method (POT) of the extreme value distributions, to model the financial tail risks associated with the empirical daily log-return distributions of the sharia-compliant stock index and three regional conventional stock markets from 01/01/1998 to 16/09/2014. These include the Dow Jones Islamic market (DJIM), the U.S. S&P 500, the S&P Europe (SPEU), and the Asian S&P (SPAS50) indexes. Using the maximum likelihood (ML) method and the bootstrap simulations to estimate the parameters of these extreme value distributions, we find a significant difference in the tail risk behaviour between the Islamic and the conventional stock markets. We find that the Islamic market index exhibits fat tail behaviour in its right tail with high likelihood of windfall profit during extreme market conditions probably due to the ban on short selling strategies in Islamic finance. However, the conventional stock markets are found to be more risky than the Islamic markets, and exhibit fatter tail behaviour in both left and right tails. Our findings suggest that during extreme market conditions, short selling strategies lead to larger financial losses in the right tail than in the left tails.

JEL Classification: G1, G13, G14.

Keywords: Tail risks, extreme value distributions, expected shortfall, BMM and POT, value at risk.

[*] Department of Economics and Econometrics, University of Johannesburg, Auckland Park, 2006, South Africa. Email: johnmu@uj.ac.za.

[**] Corresponding author. LeBow College of Business, Drexel University, Philadelphia, PA, USA. Email: hammousm@drexel.edu.

[***] Department of Economics, University of Pretoria, Pretoria, 0002, South Africa. Email: rangan.gupta@up.ac.za.

1. Introduction

Extreme episodes, better known as Black Swan events, have the worrying feature that when they occur they have great or extreme effects despite their paucity. These rare events exist in economics, finance, ecology, earth sciences and biometry, among others. However, in economics and finance, these “worst-case” episodes have become more recurrent than before, but they kept their overwhelming consequences. Examples of financial extreme events include the Black Monday of the stock market crash that took place on October 19, 1987, the turmoil in the bond market in February 1994, the 1997 Asian currency crisis, and the 2007/2008 global financial crisis. Such crises are a major concern for regulators, financial institutions and investors because of their heavy and widespread consequences.

As a consequence, many economists and financial analysts have shown increasing interest in examining the behavior of financial markets, testing financial stress and managing risks during those events. Frank Graham (1930) indicates that drastic events such as the 1920-1923 hyperinflation in Germany offer a much better way to test competing theories than normal events. The current research hopes to do so by taking into account the impact of the recent financial crises such as the 2007/2008 global financial crisis on the risks in different financial markets by applying the extreme value theory.

This paper examines the extraordinary behavior of certain random variables specifically the seemingly different conventional and Islamic stock returns, using the recently developed models known as the extreme value theory methods which quantify risks in left and right tail distributions. During extreme financial crises, these variables are characterized by extreme value changes and have very small probabilities of occurrence. The extreme value theory relies on extreme observations to derive the tail distributions. The risk is measured more efficiently using this model than by modeling the entire distributions of the random variables. Then the link between the extreme value theory and risk management is that the EVT fits extreme quantiles better than the conventional methods for tail-heavy data. In risk management, two types of extreme value distributions are frequently used namely the generalized extreme value distribution often referred to as the block of maxima method (BMM), and the Pareto distribution referred to as the peak-over-threshold method (POT).

While these methods have been applied to conventional stock markets to model the tail risks associated with the empirical return distributions, to our knowledge only Frad and Zouari (2014) used only the POT method but not the BMM method and applied it to DJIM.

Moreover, these authors have not applied this method to compare the left (long position) and right (short position) tail risks in Islamic and regional conventional stock markets which may be ostensibly different markets. Specifically, this study models the tail risks associated with the empirical return distributions of four global financial markets which include the U.S. S&P 500 index (SP500), the S&P Europe index (SPEU), the Asian S&P index (SPAS50) and the Dow Jones Islamic market (DJIM).

Accordingly, our main objective of this study is to use both the BMM and the POT methodologies in order to model the tail risk behaviour associated with the occurrence of extreme events in the Islamic and conventional stock markets. We also consider the left and the right tails of the empirical return distribution to estimate financial losses as a result of a long or short position on these markets.

The comparison between the Islamic and conventional markets in the tail distributions is relevant and useful because the Islamic stocks are arguably viewed as a viable financial system that can endure financial crises better than the conventional system and can also be used as a diversification vehicle to reduce the risk in conventional portfolios. In essence, Islamic finance may offer products and instruments that are fortified by greater social responsibility, ethical and moral values and sustainable finance.

The Islamic and conventional markets differ in several ways (Dridi and Hassan, 2010; Chapra, 2008; Dewi and Ferdian, 2010). First, Islamic markets prefer growth and small cap stocks, but conventional markets opt for value and mid cap stocks. Second, Islamic finance restricts investments in certain sectors (e.g. alcohol, tobacco, rearms, gambling, nuclear power and military-weapons activities, etc.). Third, unlike the conventional finance, Islamic finance also restricts speculative financial transactions such as financial derivatives like futures and options which have no underlying real transactions, government debt issues with a fixed coupon rate, and hedging by forward sale, interest-rate swaps and any other transactions involving items not physically in the ownership of the seller (e.g., short sales). Therefore, the research contends that Islamic stock markets have low correlations and limited long-run relationships with the conventional markets, whereby they can provide financial stability and diversification. The more recent literature underlines the superiority of Islamic stock investing in outperforming conventional investments, particularly under the recent global financial crisis (Jawadi *et al.,* 2013).

The novelty of this paper is that it makes use of two extreme value distributions, namely the generalized Pareto distribution and the generalized extreme value distribution, to simultaneously model both the left and right tails of the empirical return distribution. The paper use the maximum likelihood and the bootstrap techniques to estimate the parameters of these two distributions. In addition, unlike previous studies (e.g., Longin, 1996; McNeil and Frey, 2000; Xubiao and Gong, 2009), this paper provides reliable confidence intervals within which the tail risk measures are expected to be found. These confidence intervals are vital in assessing the investor's risk tolerance level. For example, a risk lover investor is likely to have a risk measure that is close to the upper bound of the confidence interval, while a risk averse investor is expected to be near the lower bound of the confidence interval.

The results show that the POT method generates more elaborate estimates of the shape parameters than those suggested by the maximum likelihood (ML) and the bootstrap simulation methods. They also provide evidence that the US SP500 and the Eurozone SPEU exhibit fat tail behavior in their right tails, whereas the Islamic DJIM, the Asian SPAS50, and the Eurozone SPEU exhibit fatter tail behaviour in their left tails. In addition, the paper attempts to answer the question of whether Islamic market is different from conventional market during extreme market conditions. Applying the single the analysis of variance (ANOVA) technique to the tail distribution data, we find that the Islamic DJIM market is significantly different from the conventional markets, which is likely to be due to its Sharia rules.

The paper is organized as follows. After this introduction, Section 2 presents a review of the literature on Islamic stock markets and the use of extreme value theory (EVT) distributions in finance. Section 3 discusses the modeling of extreme events using the BMM and the POT methods. Section 4 presents the empirical analysis while section 5 concludes the paper

2. Literature review

Many studies have used the EVT to measure the downside risk for conventional markets but to our knowledge this theory has not been applied to a comparison between conventional and Islamic stock markets. The EVT is becoming popular for its ability to focus directly on the tail of the empirical return distribution, and therefore it performs better than other theoretical distributions in predicting extreme events (Dacorogna et al., 1995). To reflect the volatility dynamics in the tail risk estimation, McNeil and Frey (2000) used a GARCH process with EVT and find quite interesting results that favors the extreme value theory. Other studies on EVT-based tail risk estimation include among others Gençay and Selçuk (2004), who investigated the relative performance of market risk models for the daily stock market returns of nine different emerging markets. They use the EVT to generate tail risk estimates. Their results indicate that the EVT-based tail risk estimates are more accurate at higher quantiles. Using U.S. stock market data, Longin (2005) shows how EVT can be useful to know more precisely the characteristics of the distributions of asset returns, and finally help to select a better model by focusing on the tails of the distribution. A survey of some major applications of EVT to finance is provided by Rocco (2011).

The literature on Islamic finance can be divided into four categories. These include the characteristics of Islamic finance, the relative performance of this financial system in comparison to that of other socially responsible and faith-based investments, possible links between Islamic banks and markets and their conventional counterparts, and the potential performance between the two business systems during the global crisis and the shrinking gap between them. Therefore, this review is conducted on the basis of these four themes.

The early literature deals with the unique characteristics of the Islamic financial system, particularly the prohibitions against the payment and receipt of interest. It also deals with the Islamic industry screens that restrict investment in businesses related to the sharia-forbidden activities (Abd Rahman, 2010; Bashir, 1983; Robertson, 1990; Usmani, 2002; Iqbal and Mirakhor, 2007 among others).

The more recent strand of the literature investigates the links between Islamic and conventional financial markets in terms of relative returns and relative volatility. The comparison also focuses on the relative performance during the recent global financial crisis and relies on some characteristics of Islamic markets. The markets are represented by indexes from different regions where some are a subset of the Dow Jones indexes, while others belong to the FTSE indexes, among others. The available data series of the indexes related to individual Muslim countries are not comprehensive and short in length. The literature also uses different methodologies to achieve the stated goals, ranging from the traditional linear autoregressive models to more sophisticated nonlinear models and tests (Ajmi *et al.,* 2014; Hakim and Rashidian, 2002; Dewandaru et al.,2013; Boubaker and Sghaier, 2014).

More recently, Dania and Malhotra (2013) find evidence of a positive and significant return spillover from the conventional market indexes in North America, European Union, Far East, and Pacific markets to their corresponding Islamic index returns. Sukmana and Kholid (2012) examine the risk performance of the Jakarta Islamic stock index (JAKISL) and its conventional counterpart Jakarta Composite Index (JCI) in Indonesia using GARCH models. Their result shows that investing in the Islamic stock index is less risky than investing in the conventional counterpart.

Girard and Kabir (2008) compare the differences in return performance between Islamic and non-Islamic indexes. After controlling for the firm, market and global factors, the authors do not find significant differences in terms of performance between these types of investments. Hashim (2008) examines the effect of adopting Islamic screening rules on stock index returns and risk, using monthly data from FTSE Global Islamic index. The results show that the performance of the FTSE Global Islamic is superior to that of the well diversified socially responsible index, the FTSE4Good.

The literature also explores the potential importance of Islamic finance, particularly during the recent global financial crisis. Chapra (2008) indicates that excessive lending, high leverage on the part of the conventional financial system and lack of an adequate market discipline have created the background for the global crisis. This author contends that the Islamic finance principles can help introduce better discipline into the markets and preclude new crises from happening. Dridi and Hassan (2010) compare the performance of Islamic banks and conventional banks during the recent global financial crisis in terms of the crisis impact on their profitability, credit and asset growth and external ratings. Those authors find that the two business models are impacted differently by the crisis. Dewi and Ferdian (2010) also argue that Islamic finance can be a solution to the financial crisis because it prohibits the practice of *Riba*. Ahmed (2009) claims that the global financial crisis has revealed the misunderstanding and mismanagement of risks at institutional, organizational and product levels. This author also suggests that if institutions, organizations and products had followed the principles of Islamic finance, they would have prevented the current global crisis from happening.[1] More recently, Jawadi et al. (2014) measure financial performance for Islamic

[1] There is also a growing literature on Islamic banks (see for example, Cihak and Hesse, 2010; Abd Rahman, 2010; Hesse *et al.,* 2008). Sole (2007) also presents a "good" review of how Islamic banks have become increasingly more integrated in the conventional banking system.

and conventional stock indexes for three regions (the U.S., Europe and the World) before and after the subprime crisis and point to the attractiveness of performance of Islamic stock returns, particularly after the subprime crisis. Arouri *et al.* (2011) pursue a different approach. While comparing the impacts of the financial crisis on Islamic and conventional stock markets in the same three global areas and finding less negative effects on the former than the latter, these authors examine diversified portfolios in which the Islamic stock markets outperform the conventional markets. They demonstrate that diversified portfolios of conventional and Islamic investments lead to less systemic risks.

To our knowledge, only Frad and Zouari (2014) use the EVT -POT method and apply it to DJIM to identify the extreme observations that exceed a given threshold for this index. Our study uses both the BMM and POT methods to examine the tail risk for the Islamic and regional conventional stock markets.

3. Methodology

The process of fitting log-returns series to the extreme value distributions is described below. We will discuss both the BMM and POT methods.

3.1. The Block of Maxima

Let X_1, X_2, …, X_n be a sequence of *iid* random variables representing negative returns for the left tail (or positive returns for the right tail) of the distribution of a portfolio with common density function *F*. In what follows, fluctuations of the sample maxima (minima) are investigated. Let $R_1 = X_1$ be the largest rate of return in the portfolio; and $R_m = \max(X_1, X_2, ..., X_n)$ the maximal returns or *maxima* for the right tail of the same portfolio. Corresponding results for the minima (left tail) can be easily obtained by changing the sign of the maxima into negative:

$$\min(X_1, X_2, ..., X_n) = -\max(-X_1, -X_2, ..., -X_n) \quad (1)$$

Assuming that the maxima (minima) are independent and identically distributed, we obtain the density function as follows:

$$\Pr ob(R_m \leq x) = \Pr ob(X_1 \leq x, X_2 \leq x, ..., X_n \leq x) = F(x) \times F(x) \times \cdots \times F(x) = F^n(x); \quad \forall x \in R, \; n \in N \quad (2)$$

where F(x) is cumulative distribution function of the random variable x.

Following Embrechts, Kluppelberg, and Mikosch (1997), extreme events happen in the tail of the empirical distribution. Therefore, the asymptotic behavior of the extreme returns/losses R_m must be related to the density function in its right-hand tail for positive returns or in its left hand tail for maximum/largest losses. If the series of maximum/largest losses of a portfolio during each quarterly or yearly block are centered with a mean d_n and standard deviation c_n , then its density function can be expressed as:

$$\text{Prob}\left(\left(\frac{R_m\text{-}d_n}{c_n}\right) \le x\right)=\text{Prob}(R_m \le u_n)=F(u_n) \qquad (3)$$

where $u_n = u_n(x) = c_n x + d_n$, $F(u_n)$ is the limit distribution of R_m, while d_n and c_n are the location and scale parameters, respectively. Given some continuous density function *H* such that $\frac{R_m\text{-}d_n}{c_n}$ converges in distribution in *H*, Embrechts *et al*. (1997) show that *H* belongs to the type of one of the following three density functions:

Fréchet:
$$\varphi(x) = \begin{cases} 0, \text{ for } x \le 0 & \\ & \forall \alpha > 0 \\ \exp(-x^{-\alpha}), \text{ for} & x > 0 \end{cases} \qquad (4)$$

Weibull:
$$\phi(x) = \begin{cases} \exp(-(-x)^{\alpha}), \quad x \le 0 & \\ & \alpha > 0 \\ 1, \qquad\qquad x > 0 & \end{cases} \qquad (5)$$

Gumbel:
$$\psi(x) = \exp(-e^{-x}), \qquad x \in R \qquad (6)$$

The density functions are called standard extreme value distributions.

3.1.1. Generalised extreme value distribution

Let X be a vector of extreme returns representing the maximum returns (positive or negative) of each quarterly or yearly block period as depicted in Figure 1 below, and denote by *F*, the density function of X. The limiting distribution of the normalised maximum returns X is known to be the *generalised extreme value distribution*.

PLACE FIGURE 1 HERE

Figure 1 shows the hypothetical returns for a long position on the SP500 index during five consecutive years. The maximum returns of each year block denoted by X_2, X_5, X_7, X_{11} and X_{13} have a limiting distribution known as the generalised extreme value distribution expressed as:

$$H_{(\xi,\mu,\sigma)}(x) = \exp\left\{-\left(1+\xi\frac{x-\mu}{\sigma}\right)\right\}^{-1/\xi} \tag{7}$$

ξ represents the shape parameter of the tail distribution, μ its location, and σ its scale parameter. When $\xi = \alpha^{-1} > 0$ Equation (7) corresponds to the Fréchet distribution, when $\xi = \alpha^{-1} < 0$ Equation (7) corresponds to the Weibull distribution, and when $\xi = 0$ Equation (7) corresponds to the Gumbel distribution, as shown in Equations (4), (5) and (6), respectively. Following Gilli and Kellezi (2006); we re-parameterise the generalised extreme value distribution above in order to include a tail risk measure which is referred to as the "return level":

$$H_{(\xi,\sigma,R^K)}(x) = \begin{cases} \exp\left\{-\left[\frac{\xi}{\sigma}\left(x-R^k\right)+\left(\log\left(1-\frac{1}{k}\right)\right)\right]^{-\xi}\right\}^{-1/\xi} & ;\forall\xi \neq 0 \\ \left(1-\frac{1}{k}\right)^{\exp\left(-\frac{x-R^k}{\sigma}\right)}; & \forall\xi=0 \end{cases} \tag{8}$$

where R_n^k represents the *return level* that is the maximum loss expected in one out of k periods of length n computed as:

$$R_n^k = H^{-1}_{\xi,\mu,\sigma}\left(1-\frac{1}{k}\right) \tag{9}$$

The ML method is used to estimate the parameters of the re-parameterised generalised extreme value distribution as well as their corresponding confidence intervals by maximising its log-likelihood function:

$$L\left(R^k\right) = \max_{\xi,\sigma} L\left(\xi,\sigma,R^k\right) \tag{10}$$

These confidence intervals satisfy the following condition:

$$L\left(R^{k}\right)-L\left(\hat{\xi},\hat{\sigma},\hat{R}^{k}\right)>-\frac{1}{2}\chi^{2}_{1-\alpha} \tag{11}$$

where $\chi^{2}_{1-\alpha}$ is the $(1-\alpha)^{th}$ quantile of the Chi-square distribution with 1 degree of freedom.

3.2. The peak over the threshold approach

3.2.1. Generalised Pareto distribution

Let X be a vector of extreme returns larger than a specific threshold u as depicted in Figure 2 below, and assume that the density function of X is given by F. The limiting distribution of the extreme returns above a specific threshold is known as the generalised Pareto distribution. The excess density function of X over the threshold u is defined as;

$$F_{u}(x)=\Pr ob\big((X-u)\le x / X>u\big)=\frac{F(x+u)-F(u)}{1-F(u)};\ x\ge 0 \tag{12}$$

This function is obtained via the generalised Pareto distribution in what is termed as the "peak-over-threshold method. Figure 2 illustrates how the generalised Pareto distribution fits the extreme returns above a specific threshold value of $u=3$.

PLACE FIGURE 2 HERE

This figure shows a hypothetical extreme return distribution marked as 1, 2, 3, 4, 5, 6, and 7 observed during the first half of January, and the y-axis reports their magnitudes. Assume that the return marked as 3 is our threshold. In this case the returns marked as 4, 5, 6 and 7 are considered here as extreme returns since they are larger than the threshold $u=3$. The limiting distribution of these extreme returns over the threshold $u=3$ is known as generalised Pareto distribution (GPD) and is given by the following expression:

$$G_{\xi,\beta(u)}(x)=\begin{cases}1-\left(1+\xi\dfrac{x}{\beta(u)}\right)^{-\frac{1}{\xi}}; & \xi\neq 0\\ 1\text{-exp}\left(-\dfrac{\text{x}}{\beta(\text{u})}\right); & \xi=0\end{cases} \tag{13}$$

where ξ is the shape, and u the threshold parameter, respectively. It is assumed that the random variable x is positive and that $\beta(u) > 0;\ x \geq 0$ for $\xi \geq 0$ and $0 \leq x \leq -\frac{\beta(u)}{\xi};$ for $\xi < 0$.

The shape parameter ξ is independent of the threshold u. If $\xi > 0$ then $G_{\xi,\beta(u)}$ is a Pareto distribution, while if $\xi = 0$ then $G_{\xi,\beta(u)}$ is an exponential distribution. If $\xi < 0$ then $G_{\xi,\beta(u)}$ is a Pareto type II distribution. These parameters are estimated by making use of the ML method. Firstly an optimal threshold is chosen using the mean excess function plot method introduced by Davidson and Smith (1990). The mean excess function plots the conditional mean of the extreme returns above different thresholds; the empirical mean excess function is defined as:

$$me(u) = \frac{\sum_{i=1}^{N_u} (x_i - u)}{\sum_{i=1}^{N_u} I_{u(x_i > u)}} \tag{14}$$

where $I_u = 1$ if $x_i > u$ and 0, otherwise. N_u is the number of extreme returns over the threshold *u.* If the empirical mean excess function has a positive gradient above a certain threshold *u*, it is an indication that the return series follows the GPD with a positive shape parameter ξ. In contrast, an exponentially distributed log-return series would show a horizontal mean excess function, while the short tailed log-return series would have a negatively sloped function. The parameters of the generalised Pareto distribution are obtained by maximising the following log-likelihood function:

$$L(\xi,\beta) = -N_u \mathrm{Log}(\beta) - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{N_u} \mathrm{Log}\left(1 + \frac{\xi x_i}{\beta}\right) \tag{15}$$

Embrechts, Klüppelberg and Mikosch (1997) show that the tail distribution of the generalised Pareto distribution can be expressed as follows:

$$\hat{F}(x) = 1 - \frac{N_u}{n}\left(1 + \hat{\xi}\frac{(x-u)}{\hat{\beta}}\right)^{-\frac{1}{\hat{\xi}}} \tag{16}$$

3.3. Computing tail risk measures

Although widely used to measure market risk, the value at risk (VaR) method is not a coherent measure of risk because it doesn't satisfy the sub-additivity condition. Assume that we have a long position in two financial assets z_1 and z_2, then sub-additivity means the total risk of a portfolio of these two assets must be less than the sum of the individual asset risks. Consequently, VaR doesn't satisfy the diversification principle. A more coherent risk measure is the Expected Shortfall (ES). The ES measures the expected loss of a portfolio, given that the VaR is exceeded. In this paper, we compute the VaR as the alpha quantile of the tail distribution in Equation (16), and obtain the ES by adding to the VaR the mean excess function over the VaR (see Coles, 2001 for derivation):

$$VaR(p) = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{1-p}{N_u / n}\right)^{-\hat{\xi}} - 1\right) \tag{17}$$

$$ES(p) = E\left(Y / Y > VaR(p)\right) = VaR(p) + E\left(Y - VaR(p) / Y > VaR(p)\right) \tag{18}$$

$$ES(p) = \frac{VaR(p)}{1-\hat{\xi}} + \frac{\hat{\beta} - \hat{\xi}u}{1-\hat{\xi}} \tag{19}$$

where p is the significance level at which the VaR is computed. For example, when $p = 0.99$ Equations (17) and (18) produce the tail risk measures at the 99 significance level.

4. Empirical results

4.1. Data description

We make use of closing daily stock market indexes for the Sharia-compliant stocks in the Dow Jones stock index universe and for stocks in three main regions: the United States, Europe and Asia in the S&P universe (see, for example, Hammoudeh et al., 2014; Hammoudeh et al., forthcoming). As indicated earlier, the four Islamic and regional conventional market indexes under consideration are the US SP500, the Eurozone SPEU, the Asian SPAS50 and the Islamic market DJIM. The time series for the four stock market indexes are sourced from Bloomberg. The DJIM index represents the global universe of investable equities that have been screened for Sharia compliance. The companies in this index pass the industry and financial ratio screens. The regional allocation for DJIM is classified as follows: 60.14% for the United States; 24.33% for Europe and South Africa; and 15.53% for Asia. The S&P Euro (SPEU) is a sub-index of the S&P Europe 350 and includes all Eurozone domiciled stocks from the parent index. This index is designed to be reflective of the Eurozone market, yet efficient to replicate. The Asian SPAS50 is an index that represents the most liquid 50 blue chip companies in four Asian countries: Hong Kong, Korea, Singapore, and Taiwan.

The data spans from 01/01/1998 to 16/09/2014, making a total of 4358 observations which include the recent global financial crisis period. Our aim is to model the tail distribution of these financial markets which follow different business models and compute the corresponding left and right risk measures. The left tail represents the losses for an investor with a long position on the market indexes, whereas the right tail represents the losses for an investor being short on the market indexes. Table 1 exhibits the basic statistics of the log-returns. It shows that the Asian market SPAS50 has on average the highest historical rate of return which is equal to 0.0305%, with a corresponding standard deviation of 1.47%. The Islamic market (DJIM) has the lowest historical average rate of return, with the corresponding lowest standard deviation of 1.0743%.

PLACE TABLE 1 HERE

A risk-reward analysis exhibited in Figure 3 shows that the Islamic market represented by the DJIM index has the lowest annualised risk of all the markets, and has an annualised rate of return higher than that of the US and the Euro zone markets which are represented by the SP500 and SPEU, respectively. However, the Asian market provides the annualised rate of return with a corresponding relatively higher level of risk. Unlike the Islamic markets, the Asian market is characterised by higher uncertainty and political instability that requires higher premium.

PLACE FIGURE 3 HERE

4.2. Tail estimation results

Since we are interested in both the downside (left tail) and the upside (right tail) risk measures, we collect all negative and positive log-returns, respectively, and fit them separately to the generalised extreme value distribution using the BMM method and to the

Pareto distribution using the POT method. For the generalised extreme value distribution, we first divide our sample period into quarterly blocks[2] and collect the maximum positive return (for the right tail) and the largest loss (for the left tail) of each quarterly block. The limiting distribution of these maximums (minimums) is known as the generalised extreme value distribution, whose re-parameterised version that is expressed in Equation (8) is used to estimate the shape and scale parameters using the maximum likelihood (*ML*) method. Table 2 reports the *ML* estimates of these parameters as well as their confidence intervals. For the purpose of robustness, we report the best estimate and its corresponding bootstrapped value.

PLACE TABLE 2 HERE

Table 2 reports the shape (ξ) and the scale (σ) parameters of the re-parameterised GEV function shown in Equation (8), the point estimates and their corresponding confidence intervals for the Islamic and conventional stock market at the 1% and 5% significance levels. The maximum likelihood estimates are referred to as *ML,* whereas the bootstrapped estimates are denoted by *BS*. Moreover, *LT* (*RT*) refers to the left tail (right tail) of the empirical return distribution, representing the downside risk and upside risk, respectively. We find that the BMM method generates only positive shape parameters for all of the four market indexes used in our study. A positive shape parameter is an indication that these market indexes have fatter tails than the normal distribution. The quantile-quantile plots shown Figures 6, 7, 8 and 9 confirm that the generalised extreme value distribution best fits the set of quarterly block maximums (minimums) data.

Given the parameters of the re-parameterised generalized extreme value distribution, we thereafter compute one tail risk measure associated with the generalised extreme value distribution, namely the *return level* (see Gilli and Kellezi, 2006). We denote by *RL* the return level which represents the maximum loss expected in one out of ten quarters. Table 3 reports the *RL* for both the left and the right tails of the empirical distribution at the 1% and 5% significance levels as per the Basel II accord.[3] Their confidence intervals are reported in

[2] One of the criticisms of the BMM method is that there is not a standard way of grouping data in blocks of maxima. Given the length of our daily sample period (i.e., 16 years), we believe that grouping the maximums (minimums) in quarterly blocks would result in enough data points to generate unbiased estimates of the generalised extreme value distribution.

[3] The Basel II accords recommend that the VaR be estimated at higher quantile, i.e., the 1% significance level for the next 10 trading days.

Tables 8 and 9. For the purpose of robustness, we also report the bootstrapped return level after 1000 resamples.

PLACE TABLE 3 HERE

For example, using the US SP500 market index, one would say that at 1% significance level the maximum loss observed during a period of one quarter exceeds 4.8% in one out of ten quarters on average for an investor with a long position on the market index (left tail). Figure 4 below highlights the differences in the return level of each market index at both the 1% and 5% significance levels. At these levels, we find that due to its Sharia laws, the Islamic market is less risky than other three market indexes. Both the left and right *ML* and bootstrapped maximum losses during one quarter are expected to exceed 3.8% on average in one out of ten quarters for an investor with long and/or short positions in the Islamic market. In contrast, the Asian market index SPAS50 is more risky than the rest of the market indexes in our portfolio. Its maximum loss observed during one quarter exceeds 5.8% in one out of ten quarters on average for an investor with a long position on the index (left tail) and 6% for an investor with a short position on the index (right tail).

PLACE FIGURE 4 HERE

Based on the specific market regulations, we find that in the US market and the Sharia - law compliant market which has 64% of it constiutents in the US maket, the portfolio risk measure is indepedend of the investment strategy used, i.e., the long or the short position. The maximum expected losses in these markets are almost the same for both the long position (left tail) and short positions (right tail) on the market indexes. However, in the Eurozone and Asian markets, we find that the short (selling) position generates higher risk than the long only position. We argue that this has to do with the presence of market speculations and short selling regulations, particualrly during the debt crisis.

Contrary to the BMM methodology, the POT methodology produces more reliable and efficient shape parameters, and seems to be well suited for the modelling of the tails of financial time series (see for example Coles, 2001; McNeil, Frey and Embrechts, 2005 for more documentation of this result). The POT methodology proceeds as follows. Firstly, an optimal threshold[4] value is determined by using the mean excess function method which is described above. We report in Figures 6, 7, 8, and 9 the plots of the mean excess function, the excess distribution and the quantile-quantile distribution for the left tail of the empirical distribution. A visual analysis suggests that the optimal threshold value for the four market indexes varies between 3% and 5%. These values are located at the beginning of a portion of the sample mean excess plot that is roughly linear. Given the large number of values the thresholds can take in this interval of 3% to 5%, and the resulting subjectivity about the correct threshold value, in this study we follow Mackay, Challenor, and Bahaj (2010), Damon (2009); and Sigauke, Vester and Chikobvu (2012) who suggest the preferable use of the 90^{th} quantile of the empirical return distribution[5].

We follow the same procedure described above for the BMM method to separate the data for the left and right tails, respectively. Using the LM estimation method, we obtain the shape and scale parameters of the generalised Pareto distribution expressed in Equation (13). We also make use the Bonferroni confidence interval to correct for the sample bias. Two types of confidence intervals are reported: the *ML* confidence interval and the Bonferroni confidence interval for the left and the right tail distributions at the 1% and 5% significance levels, respectively. The *ML* and bootstrapped point estimates are reported in Table 4 in the column labelled "best estimate". For example, using the SP500 one would say that the 1% level, *ML* and bootstrapped estimates of the left tail shape are 0.397% and 0.011%, respectively. Their corresponding confidence intervals are -0.034 (-1.028), and 1.669 (3.99), respectively. These numbers represent the smallest and the largest values these parameters can take.

Unlike the BMM methodology which produced only positive shape parameters, the POT methodology produces negative and positive shape parameters. The negative shape parameter indicates that the tail of the empirical distribution (left and/or right) is thinner than

[4] The mean excess analysis may be used to select an optimum threshold. An optimal threshold is crucial to obtaining a reliable risk measures. Notice that a lower threshold is likely to reduce the variance of the estimates of the Generalised Pareto Distribution and induce a bias in the data above the threshold. A higher threshold reduces the bias but increases the volatility of the estimate of the GPD distribution. See for example Danielsson and de Vries (1997) and Dupuis (1998) for more discussion on this issue. To avoid these issues, we use the 90^{th} quantile of the empirical log-return distribution as the threshold value.

[5] For more discussion on the choice of the optimal threshold value, we refer the interested readers to the following studies Damen (2009); Mackay et al. (2010); Sigauke at al. (2012).

the tail of the normal distribution. However a positive shape parameter is an indication that the empirical distribution (left and/or right) has a fatter tail than that of the normal distribution, which can lead to the occurrence of extreme losses. Table 4 shows that the US SP500 and the Eurozone indexes have thinner right tail distributions, meaning that the probability of the occurrence of extreme losses due to short (selling) positions is minimal. However, on the downside, the Eurozone SPEU, the Asian SPAS50, and the Sharia-based Islamic market indexes exhibit negative shape parameters, meaning that the probability of extreme losses due to long position is minimal. These results highlight the importance of the generalised Pareto distribution in fitting appropriately the tails of time series data characterised by extreme events.

PLACE TABLE 4 HERE

Based on these estimates, we compute two types of risk measures: the VaR and the ES. Theoretically, the ES is equal to the sum of VaR and the average of all losses exceeding the VaR. Therefore, we expect in all cases the VaR estimates to be of less magnitude than the ES estimates. Table 5 reports the risk measures for both the left and right tail distributions. Their confidence intervals are reported in Tables 8, 9, 10, and 11 . We find almost the same results with the BMM methodology, except for the US SP500 market index which results in the two largest risk measures, i.e. 17.15% (VaR) and 27.15% (ES) at the 5% significant level. We believe that this has to do with the recent 2008 – 2009 financial crisis.

PLACE TABLE 5 HERE

Figure 5 highlights the differences in the magnitude of the risk measures correponding to each of the four market indexes. Although the Sharia-based Islamic market index (DJIM) remains the least risky market, the POT methodology highlights the relatively high risk associated with the short (selling) position in the right tail of the return distributions of conventional markets. In general, the short positions lead to a higher likelihood of the occurrence of maximum/extreme losses.

PLACE FIGURE 5 HERE

In addition, we attempt to answer the question of whether the Islamic market as represented by the DJIM is different from the three conventional financial markets. We apply the ANOVA technique to the tail distribution data, i.e. the quartely maximum and nimum return series. Our aim in this section is to study the variability (dynamics) of each stock market during extreme events. In other words, we attempt to see whether the variability of the Islamic market during extreme market conditions is the same as that of conventional stock markets. We therefore test the null hypothesis of equal variability for the four markets i.e. *H0: V1=V2=V3=V4* against H1: at least one stock market different from the others, where *V1* is the variability in the SP500 market, *V2* is the variability in the SPEU market, *V3* is the variability in the SPAS50 market and *V4* is the variability in the Islamic DJIM market.

Two results can be obtained from this test. First, if we fail to reject the null hypothesis H0, it means that there is no difference between the Islamic and the conventional markets during extreme market events. Second, if we reject the null hypothesis it means that at least one market is different from others. In this case, we need to further test two sets of the null hypotheses:

i. The conventional stock markets are not different (they have equal variability during extreme events) against the alternative that they are different. We refer to these hypotheses as *H01*: *V1=V2=V3*, and *H11:* at least one conventional market is different from the rest.

ii. The Islamic stock market is different from each one of the conventional market; in this case the following hypotheses are formulated: *H02: V4=V1* against *H12: V4≠V1*; and *H03: V4=V2* against *H13: V4≠V2*; and *H04: V4=V3* against *H14: V4≠V3*. With *V1, V2, V3,* and *V4* defined as above.

Tables 6 and 7 report the test statistic corresponding to each hypothesis test as well as its p-values. We reject the null hypothesis *H0* of equal variability in all stock markets and conclude that at least one stock market is different from the others. To find out which one it is, we first test the null hypothesis *H01* of equal variability in all conventional stock markets. We fail to reject this null hypothesis only at 10% significance level and conclude that the variability in convnentional stock markets during extreme events are the same. Lastly, we test the null hypothesis of equal variability between the Islamic market and each one of the conventional stock markets; that is hypotheses *H02, H03*, and *H04*. We do reject these null hypotheses at the 5% signifiance level for *H03* and *H04*, and at the10% significance level for

H02; and conclude that the Islamic DJIM market is significantly different from the convnentional stock markets.

5. Conclusion

This paper makes use of two techniques used in the extreme value theory, namely the block of maxima method (BMM) based on the generalised extreme value dostribution and the peak-over-the threshold method (POT) based on the generalised Pareto distribution, in order to model the tails of the empirical distributions of three regional conventional imdxes and the Islamic market indexes. They indexes are represenetd by the US SP500, the Eurozone SPEU, the Asian SPAS50 and the Islamic DJIM. The main objective of the paper is to compute the financial tail risk measures associated with the distributions of these markets which follow different business models. To achieve this purpose, the study bigins by separating the log-return data for the left and the right tail distributions.

For the BMM method, the paper groups the log-return data in 67 independent and overlapping quarterly blocks and identified the minimum (maximum) of each block as the adequate inputs to the BMM methodology. However, the inputs for the POT methodology have been identified as the excesses over the threshold of the 90^{th} quantile of the empirical log-return distribution.

Using the ML method and the 1000 bootstrap simulations, we estimate the parameters of the re-parameterised generalised extreme value distribution. The estimation of the re-parameterised distribution results in positive shape parameters for all four market indexes, leading to the conclusion that the BMM method suggests that these market indexes exhibit fatter tails than the tail of the normal distribution. However, when the POT methodology is used, we find more elaborate estimates of the shape parameters. We find that the US SP500 and the Eurozone SPEU exhibit fatter tail behaviour in the right tail, whereas the Islamic DJIM, the Asian SPAS50, and the Eurozone SPEU exhibit fatter tail behaviour in the left tails. Stock markets with fatter left tails are prone to higher risk due to short selling positions. However, based on the risk-reward analysis reported in Fgure 3, we find that the Islamic market, although exhibiting a fatter left tail behaviour, is less risky than the conventional stock markets. Since short selling and other excessive risk taking behaviours are not allowed in Islamic markets, we argue that its left fat-tailedness behaviour is an indication of windfall profits on long positions only that investors can reap during extreme events. We have applied the ANOVA technique to the tail distribution data in order to determine whether the Islamic market is different from the conventional markets. Using different statistical tests, we find that the Islamic stock market is indeed significantly different from the conventional stock markets during extreme market events. We therefore recommend the Islamic finance as a solution to financial crises in order to curb excessive risk taking behaviour in conventional stock markets

Based on the shape parameters, we find that the Asian SPAS50 market index is the more risky market in its right tail than in its left tail. This is an indication that short (selling) positions on this market index have a more negative impact on its performance. The Islamic market index is the least risky market index most likely due to its restrictive Sharia laws that discourage high risk taking behaviour. However, the developed markets (the US SP500, and the Eurozone SPEU) are relatively riskier.

The results of this current study are really significant because they show clearly that during major crises the Islamic stock index is not only less risky but also significantly different from the conventional markets. Thus, the results come differently to those of the recent studies which show that the former is no different from the counventional counterparts in different regions. Both the left and right risk measures depend on whether the investor is long or short on these market indexes. In general, we find that in most volatile and worst market conditions, short (selling) positions on conventional stock market indexes have a more negative impact on the respective portfolio performance than long position strategies. Finally, Islamic market provides generous opportunities for windfall profits during periods of financial crises.

References

Abd Rahman, Z., (2010). *Contracts & The Products of Islamic Banking*, Kuala Lumpur, MY: CERT Publications.

Ahmed, H., (2009). Financial crisis risks and lessons for Islamic finance, *International Journal of Islam Finance* 1, 7-32.

Arouri, M.H., Lihiani, A. and Nguyen, D. K. (2011). Return and volatility transmission between world oil prices and stock markets of GCC. *Economic Modelling* 26, 1815-1825.

Ajmi, A. N., Hammoudeh, S., Nguyen, D. K., Sarafrazi, S., (2014). *How strong are the causal relationships between Islamic and conventional finance systems? Evidence from linear and nonlinear tests. Journal of International Financial Markets, Institutions and Money* 28, 213-227.

Arouri, M. E., Ameur, H. B., Jawadi, N., F. Jawadi and Louhichi, W., (2013). Are Islamic finance innovations enough for investors to escape from a financial downturn? Further evidence from portfolio simulations, *Applied Economics* 45, 3412-3420.

Bashir, B. A., (1983). Portfolio management of Islamic banks, Certainty Model, *Journal of Banking and Finance* 7, 339-354.

Boubaker, H., and Sghaier, N., (2014). *Semiparametric generalized long memory modelling of GCC stock market return: a wavelet approach. IPAG*, Working Paper 066. Available on the link: http://www.ipag.fr/wp-content/uploads/recherche/WP/IPAG_WP_2014_066.pdf

Cihak, M., Hesse, H., (2010). Islamic Banks and Financial Stability: An Empirical Analysis, Journal of Financial Services Research, *Journal of Financial Services Research*, 38(2), 95-113.

Coles, S. (2001). *An Introduction to Statistical Modeling of Extreme Values,* Springer Series in Statistics. Springer, London

Dacorogna, M. M., Müller, U. A., Pictet, O. V., DeVries, C. G., (1995) *The Distribution of Extremal Foreign Exchange Rate Returns in Extremely Large Data Sets*. Preprint, O&A Research Group.

Damon, L (2009). Modelling tail behaviour with extreme value theory. *Risk management*, **17**, 13- 18

Dania, A., Malhotra, D.K., (2013), An Empirical Examination of the Dynamic Linkages of Faith-Based Socially Responsible Investing, *The Journal of Wealth Management* 16(1), 65-79.

Davison, A. C., and Smith, R. L., (1990). Models for exceedances over high thresholds. *Journal of Royal Statistical Society*, 52(3), 393–442.

Danielsson, J., and DeVries, C. G., (1997). Tail index and quantile estimation with very high frequency data. *Journal of Empirical Finance*, 4, 241 – 257,

Dewandaru, G., El Alaoui, A., Masih, A.M.A., and Alhabshi, S.O., (2013). *Comovement and resiliency of Islamic equity market: evidence from GCC Islamic equity index based on wavelet analysis.* MPRA working paper number 56980. Available on the link: http://mpra.ub.uni-muenchen.de/56980

Dewi M., Ferdian. I. R., (2010). *Islamic finance: A therapy for healing the global financial crisis*, http://ebookpdf. Net/islamic-finance-a-therapy-for-healing-the-global-financial-crisis.html, accessed on 10/04/2012.

Dridi, J., Hassan, M., (2010). *The effects of global crisis on Islamic and conventional banks: A comparative study*, International Monetary Fund Working Paper No, 10/201.

Dupuis, D.J. (1998). Exceedances over high thresholds: A guide to threshold selection. *Extremes*, 1(3), 251–261.

Embrechts, P., Klüppelberg, C. and Mikosch, T. (1997). *Modelling Extremal Events for Insurance and Finance.* Springer-Verlag, Berlin.

Frad, H., Zouari, E. (2014). Estimation of value-at-risk measures in the Islamic stock market: Approach based on Extreme Value Theory (EVT). *Journal of World Economic Research,* 3(2), 15-20. Published online July 10, (2014) (http://www.sciencepublishinggroup.com/j/jwer) doi: 10.11648/j.jwer.20140302.11

Gençay, R., and Selçuk, F., (2004). Extreme Value Theory and Value-at-Risk: Relative Performance in Emerging Markets. *International Journal of Forecasting*, 20, 287– 303

Gilli, M., and Kellezi, E. (2006). An application of extreme value theory to measuring financial risk. *Computational Economics*, 27, 207 – 228.

Girard, E.C., Kabir, M., (2008). Is there a cost to faith-based investing: Evidence from FTSE Islamic indices. *Journal of Investing* 17(4), 112-121.

Graham, F. D. (1930). *Exchanges, Prices, and Production in Hyperinflation Countries: Germany, 1920-1923*. Princeton University Press, Princeton, NJ.

Hakim, S., Rashidian, M., (2002). *Risk and return of Islamic stock market indexes*. Paper Presented at the 9th Economic Research Forum Annual Conference in Sharjah, U.A.E. on October 26-28, 2002.

Hammoudeh, S., W. Mensi, J. C. Reboredo and D. K. Nguyen (2014). Dynamic dependence of Global Islamic stock index with global conventional indexes and risk factors. *Pacific Basin Financial Journal,* 30, 189-206.

Hammoudeh, S., Kim. W.J. and S. Sarafrazi, S. (forthcoming). Sources of Fluctuations in Islamic, U.S., EU and Asia Equity Markets: The Roles of Economic Uncertainty, Interest Rates. *Emerging Markets Finance and Trade*.

Hashim, N., (2008). *The FTSE global Islamic and the risk dilemma,* AIUB Bus Econ Working Paper Series, No 2008-08. http://orp.aiub.edu/WorkingPaper/WorkingPaper.aspx?year=2008.

Hesse, H., Jobst, A. A., Sole, J., (2008). Trends and challenges in Islamic finance, *World Economics,* 9(2), 175-193.

Iqbal, Z., Mirakhor, A., (2007). *An introduction to Islamic finance: theory and practice*, John Wiley & Sons (Asia) Pte Ltd, Singapore.

Jawadi, F., Sousa, R. A., (2013). Structural breaks and nonlinearity in U.S. and UK public debts. *Applied Economics Letters* 20(7), 653-657.

Jawadi, F., Jawadi, N. et Louhichi, W. (2014), Conventional and Islamic Stock Price Performance: An Empirical Investigation, *International Economics,* 137, 73–87.

Longin, F. M. (1996). The Asymptotic Distribution of Extreme Stock Market Returns," *Journal of Business*, 69, 3, 383–408.

Longin, F. (2005). The choice of the distribution of asset returns: How extreme value theory can help? *Journal of Banking & Finance*, Elsevier, 29(4), 1017-1035.

Mackay, E. B. L, Challenor, A. S., and Bahay A. S. (2010). On the use of discrete seasonal and directional models for estimation of extreme wave conditions. *Ocean Engineering*, 37, 425 – 442.

McNeil, A. J., and Frey, R. (2000). Estimation of Tail-related Risk Measures for Heteroskedastic Financial Time Series: An Extreme Value Approach," *Journal of Empirical Finance,* 7, 271– 300.

McNeil, A., Frey, R. and Embrechts, P. (2005). *Quantitative Risk Management: Concepts Techniques and Tools.* Princeton University Press, Princeton, NJ.

Robertson, J., (1990). *Future wealth: A new economics for the 21st century*. London:

Cassell Publications.

Rocco, M. (2011). *Extreme value theory for finance: a survey*. Bank of Canada. https://ideas.repec.org/p/bdi/opques/qef_99_11.html

Sigauke, S. Verster A., and Chikobvu, E. (2012). Tail quantile estimation of the heteroskedastic intraday increases in peak electricity demand. *Open Journal of Statistics*, 2, 435 - 442

Sole, J., (2007). *Introducing Islamic banks into conventional banking systems*, IMF Working Paper, International Monetary Fund, Washington, D. C.

Sukmana, R., Kolid, M., (2012). Impact of global financial crisis on Islamic and conventional stocks in emerging market: an application of ARCH and GARCH method, *Asian Academy of Management Journal of Accounting & Finance,* to appear.

Usmani, M. T., (2002). *An introduction to Islamic finance*, The Netherlands:

Kluwer Law International.

Xubiao H., Gong, P. (2009). Measuring the Coupled Risks: A Copula-Based CVaR Model. *Journal of Computational and Applied Mathematics*, 223, 2, 1066-1080.

Table 1: Summary statistics

Index	Mean	Std Dev	Skewness	Kurtosis
SP500	0.0167	1.2582	-0.2039	7.9104
SPEU	0.0075	1.4017	-0.0994	4.4679
SPAS50	0.0305	1.4689	0.03	5.5
DJIM	0.0187	1.0743	-0.322	6.5891

Table 2: BMM shape and scale estimates[6]

			Alpha=1%			Alpha=5%		
			LOWER BOUND	POINT ESTIMATE	UPPER BOUND	LOWER BOUND	POINT ESTIMATE	UPPER BOUND
SP500	LT	MLξ	0.089	0.324	0.574	0.136	0.324	0.524
		MLσ	0.007	0.008	0.011	0.0071	0.00825	0.01
		BSξ		0.3239			0.32	
		BSσ		0.0082			0.0082	
	RT	MLξ	0.092	0.36	0.635	0.145	0.36	0.581
		MLσ	0.007	0.008	0.011	0.007	0.0082	0.01
		BSξ		0.3605			0.3604	
		BSσ		0.0082			0.0082	
SPEU	LT	MLξ	0.016	0.249	0.535	0.06	0.249	0.473
		MLσ	0.007	0.009	0.011	0.0072	0.00851	0.1034
		BSξ		0.2487			0.2487	
		BSσ		0.0505			0.00851	

[6] *LT* denotes left tail, while *RT* refers to right tail. *ML* and *BS* refer to the maximum likelihood and bootstraps estimates for the shape (ξ) and the scale (σ) parameters, respectively

MLξ: estimate of the shape parameter using the Maximum likelihood method

MLσ: estimate of the scale parameter using the Maximum likelihood method

BSξ: estimate of the shape parameter using the Bootstrap technique

BSσ: estimate of the scale parameter using the Bootstrap technique

NB: each Bootstrap technique involves 1000 simulations in order to obtain unbiased estimates

	RT	MLξ	0.089	0.326	0.554	0.137	0.356	0.51
		MLσ	0.008	0.009	0.012	0.0079	0.00921	0.1113
		BSξ		0.0326			0.326	
		BSσ		0.0092			0.009205	
SPAS50	LT	MLξ	-0.014	0.244	0.604	0.032	0.244	0.526
		MLσ	0.009	0.011	0.014	0.0091	0.01082	0.0132
		BSξ		0.2443			0.2443	
		BSσ		0.0108			0.010816	
	RT	MLξ	0.048	0.635	0.791	0.137	0.338	0.552
		MLσ	0.009	0.011	0.013	0.0088	0.01038	0.0128
		BSξ		0.3381			0.3381	
		BSσ		0.0104			0.010379	
DJIM	LT	MLξ	0.06	0.29	0.556	0.105	0.29	0.5
		MLσ	0.006	0.007	0.009	0.0058	0.00674	0.0082
		BSξ		0.2902			0.2902	
		BSσ		0.0067			0.00674	
	RT	MLξ	0.019	0.007		0.057	0.23	0.435
		MLσ	0.006	0.039		0.0063	0.0071	0.0086
		BSξ		0.0071			0.02302	
		BSσ		0.0386			0.007095	

Note. See footnotes 4 for Table 2.

Table 3: BMM return levels

		Alpha=1%		Alpha=5%	
		ML	Bootstrap	ML	Bootstrap
SP500	Left.RL	4.8	4.78	4.778	4.777
	Right.RL	4.8	4.81	4.805	4.8048
SPEU	Left.RL	5.1	5.05	5.053	5.0526
	Right.RL	5.4	5.41	5.412	5.4121
SPAS 50	Left.RL	5.8	5.82	5.824	5.824
	Right.RL	6.001	6.02	6.023	6.0227
DJIM	Left.RL	3.901	3.92	3.92	3.9195
	Right.RL	3.9021	3.86	3.862	3.8619

Table 4: POT shape and scale estimates

			Alpha=1%			Alpha=5%		
			LOWER BOUND	BEST ESTIMATE	UPPER BOUND	LOWER BOUND	BEST ESTIMATE	UPPER BOUND
SP500	L.T	MLξ	-0.034	0.379	1.669	0.026	0.379	1.217
		MLσ	0.005	0.011	0.025	0.006	0.011	0.021
		Bcaξ	-1.028	0.3787	3.99	-0.236	0.3787	2.919
		Bcaσ	0.001	0.113	0.08	0.002	0.0113	0.03
	R.T	MLξ	-0.003	-0.003	-0.178	-0.03	-0.003	-0.159
		MLσ	0.01	0.016	0.03	0.011	0.016	0.025
		Bcaξ	-1.191	-0.0032	0.035	-0.672	-0.0032	0.267
		Bcaσ	0.01	0.0163	0.32	0.011	0.016	0.023
SPEU	L.T	MLξ	-0.085	-0.085	-0.268	-0.085	-0.085	-0.246
		MLσ	0.001	0.016	0.011	0	0.016	0.012
		Bcaξ	-0.615	-0.0854	0.594	-0.378	-0.0852	0.383
		Bcaσ	0.006	0.0163	0.033	0.009	0.0163	0.027
	R.T	MLξ	-0.136	-0.136	-0.3	-0.136	-0.136	-0.283
		MLσ	0.001	0.022	0.015	0	0.022	0.017
		Bcaξ	-1.082	-0.1356	0.782	-0.467	-0.1356	0.395
		Bcaσ	0.007	0.0224	0.049	0.011	0.022	0.039
SPAS50	L.T	MLξ	-0.051	-0.051	-0.236	-0.051	-0.051	-0.21
		MLσ	0.001	0.017	0.012	0.001	0.017	0.013
		Bcaξ	-0.463	-0.0507	0.408	-0.306	-0.51	0.289
		Bcaσ	0.008	0.0166	0.031	0.011	0.011	0.029
	R.T	MLξ	0.115	0.115	0.758	0.115	0.115	0.544
		MLσ	0.011	0.017	0.029	0.012	0.017	0.025
		Bcaξ	-0.405	0.1151	1.112	-0.161	0.1151	0.817
		Bcaσ	0.007	0.0172	0.031	0.009	0.172	0.027
DJIM	L.T	MLξ	-0.06	-0.06	-0.295	-0.06	-0.06	-0.276
		MLσ	0.001	0.016	0.009	0.001	0.016	0.01
		Bcaξ	-1.599	-0.06	1.781	-1.087	-0.06	1.594
		Bcaσ	0.001	0.0161	0.08	0.02	0.0161	0.043
	R.T	MLξ	0.201	0.266	1.62	-0.016	0.266	1.113
		MLσ	0.004	0.008	0.02	0.005	0.008	0.016
		Bcaξ	0.23	0.266	1.6	-1.587	0.2664	0.886

Bcaσ	0.0031	0.008	0.021	0.002	0.0083	0.017

Table 5: POT estimates of VaR and ES

		Alpha=1%	Alpha=5%
SP500	Left.VaR	11.06	17.15
	Left.ES	17.79	27.15
	Right.VaR	8.65	10.44
	Right.ES	10.26	12.04
SPEU	Left.VaR	8.22	8.22
	Left.ES	9.31	9.31
	Right.VaR	9.47	9.47
	Right.ES	10.67	10.67
SPAS50	Left.VaR	9.08	9.1
	Left.ES	10.37	10.4
	Right.VaR	11.53	11.53
	Right.ES	14.58	14.58
DJIM	Left.VaR	7.38	7.38
	Left.ES	8.65	8.65
	Right.VaR	8.024	7.02
	Right.ES	10.18	9.61

Table 6: Summary of ANOVA tests for lower tail

Source of Variation	**Sum-of Squared**	**Degree of Freedom**	**Mean Square**	**F. Calculated**	**P value**	**F. Theoretical**
H0:V1=V2=V3=V4						
Between Markets	43.6297	3	14.54323	5.8797	0.000672	2.6388
Error	652.9923	264	2.4725			
Total	696.622	267				
Decision					**Reject H0**	
H01: V1=V2=V3						
Between Markets	12.7728	2	6.4864	2.3885	0.0944	3.0415
Error	537.7111	198	2.7157			
Total	550.6839	200				
Decision					**Do not Reject H0 @ 10%**	

H02: V4=V1						
Between Markets	6.9886	1	6.9886	2.9873	0.086258	3.912875
Error	308.804	132	2.3394			
Total	315.7926	133				
Decision					**RejectOnlyat10%**	
H03: V4=V2						
Between Markets	21.9468	1	21.9468	11.1996	0.001066	3.912875
Error	258.6684	132	1.9596			
Total	280.6152	133				
Decision					**Reject H0**	
H04: V4=V3						
Between Markets	38.8648	1	38.8648	16.2305	0.000094	3.9129
Error	316.0822	132	2.3946			
Total	354.947	133				
Decision					**Reject H0**	

Table 7: Summary of NOVA tests for upper tail

Source of Variation	**Sum-of Squared**	**Degree of Freedom**	**Mean Square**	**F Calculated**	**P value**	**F Theoretical**
H0:V1=V2=V3=V4						
Between Markets	54.9709	3	18.3236	6.0139	0.00056	2.6388
Error	804.3742	264	3.0469			
Total	859.3451	267				
Decision					**Reject H0**	
H01: V1=V2=V3						
Between Markets	20.1067	2	10.0534	2.8871	0.058087	3.041518
Error	689.4675	198	3.4822			
Total	709.5742	200				
Decision					**Do not Reject H0 @ 10%**	
H02: V4=V1						
Between Markets	5.9004	1	5.9004	2.654607	0.1056	3.912875
Error	293.3947	132	2.2227			
Total	299.295	133				
Decision					**RejectOnlyat10%**	
H03: V4=V2						
Between Markets	26.6209	1	26.6209	10.5848	0.00148	3.91288
Error	331.9807	132	2.515			
Total	358.6016	133				
Decision					**Reject H0**	
H04: V4=V3						
Between Markets	47.2605	1	47.2605	15.2598	0.000149	3.912875

Error	408.8121	132	3.0971
Total	456.0726	133	
Decision			**Reject H0**

Table 8: ML and Bca Estimates of RL when alpha=1% for BMM method

		LOW BOUND	POINT ESTIMATE		UPPER BOUND
		ML	ML	Bca	ML
SP500	Left RL	4.71	4.8	4.78	6.26
	Right RL	4.8	4.8	4.81	6.6
SPEU	Left RL	5.1	5.1	5.05	6.5
	Right RL	5.4	5.4	5.41	7.2101
SPAS50	Left RL	5.8	5.8	5.82	7.9
	Right RL	6.01	6.001	6.02	8.2
DJIM	Left RL	3.9	3.901	3.92	5.1
	Right RL	3.9	3.9021	3.86	16.4

Table 9: ML and Bca Estimates of VaR when alpha=5% for BMM method

		LOW BOUND	POINT ESTIMATE		UPPER BOUND
		ML	ML	Bca	ML
SP500	Left RL	3.968	4.778	4.777	6.377
	Right RL	3.931	4.805	4.8048	6.638
SPEU	Left RL	4.334	5.053	5.0526	6.472
	Right RL	4.503	5.412	5.4121	7.168
SPAS 50	Left RL	4.913	5.824	5.824	7.871

	Right RL	4.97	6.023	6.0227	8.17
DJIM	Left RL	3.301	3.92	3.9195	5.137
	Right RL	3.287	3.862	3.8619	4.948

Table 10: ML Estimates of VaR and ES when alpha=**1%** for POT method

		LOW BOUND	best ESTIMATE	UPPER BOUND
SP500	Left VaR	-58.94	11.06	598
	Left ES	7.12	17.79	810.37
	Right VaR	-21.35	8.65	27.75
	Right ES	7.33	10.26	120.05
SPEU	Left VaR	-21.79	8.22	28.81
	Left ES	7.15	9.31	142.3
	Right VaR	-20.53	9.47	35.33
	Right ES	8.23	10.67	182.12
SPAS 50	Left VaR	-20.92	9.08	27.32
	Left ES	7.81	10.37	78.75
	Right VaR	-38.47	11.53	55.61
	Right ES	9.32	14.58	164.5
DJIM	Left VaR	-22.62	7.38	125.74
	Left ES	0.01	8.65	183.39
	Right VaR	23.01	8.024	118.027
	Right ES	0.31	10.18	162.03

Table 11: ML Estimates of VaR and ES when alpha=5% for the POT method

		LOW BOUND	best ESTIMATE	UPPER BOUND
SP500	Left VaR	-92.85	17.15	74.254
	Left ES	0.01	27.15	248.65

	Right VaR	-29.56	10.44	28.7
	Right ES	8.75	12.04	57.61
SPEU	Left VaR	-21.78	8.22	16.47
	Left ES	7.46	9.31	32.5
	Right VaR	-20.53	9.47	19.48
	Right ES	8.6	10.67	38.03
SPAS50	Left VaR	-20.9	9.1	17.4
	Left ES	8.1	10.4	30.9
	Right VaR	-38.47	11.53	29.22
	Right ES	9.95	14.58	88.41
DJIM	Left VaR	-22.62	7.38	28.22
	Left ES	0.01	8.65	183.39
	Right VaR	-22.98	7.02	27.21
	Right ES	0.01	9.61	257.14

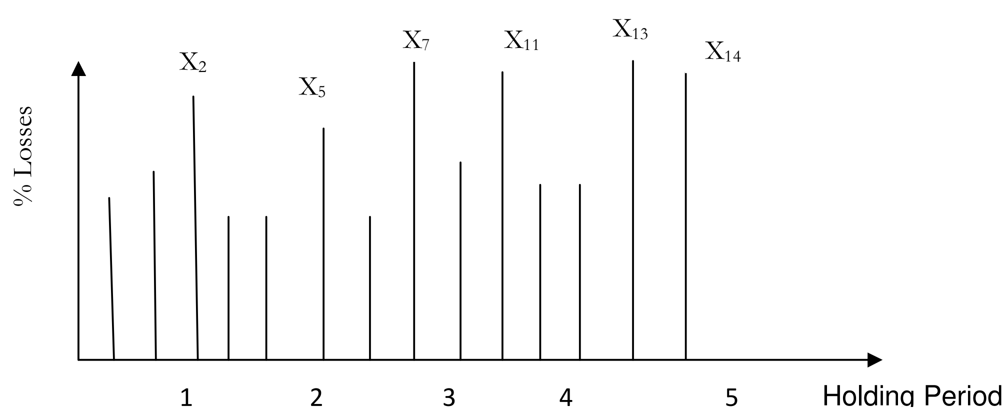

Figure 1. Hypothetical return series for a long position on the SP500 index during years 1, 2, 3, 4, and 5.

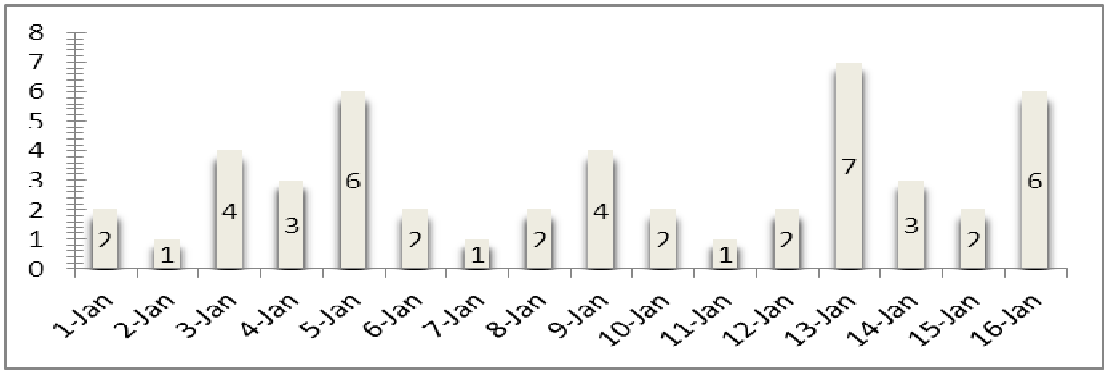

Figure 2: A hypothetical extreme return distribution with a threshold $u = 3$.

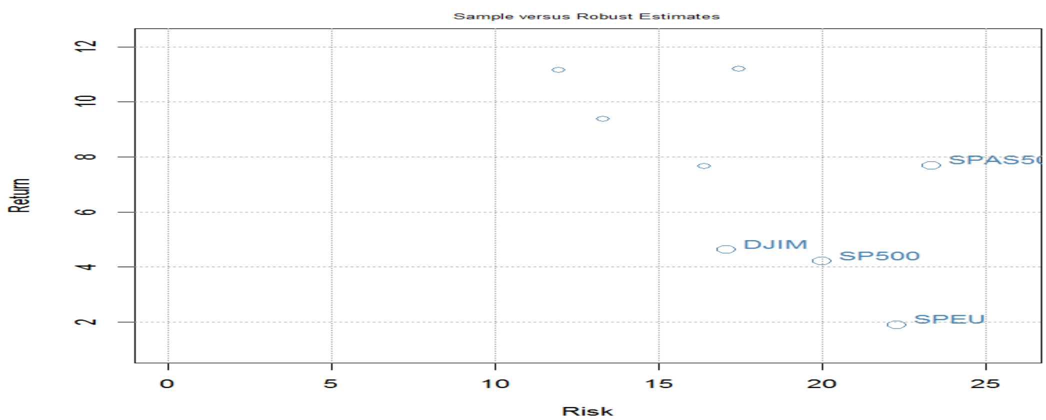

Figure 3: Risk-reward plot.

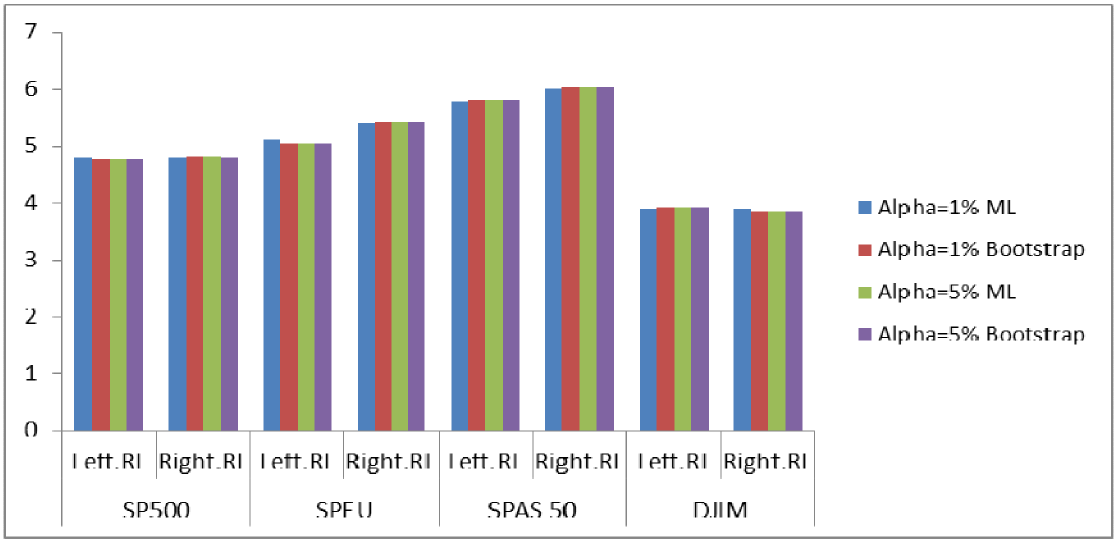

Figure 4: BMM comparative return levels.

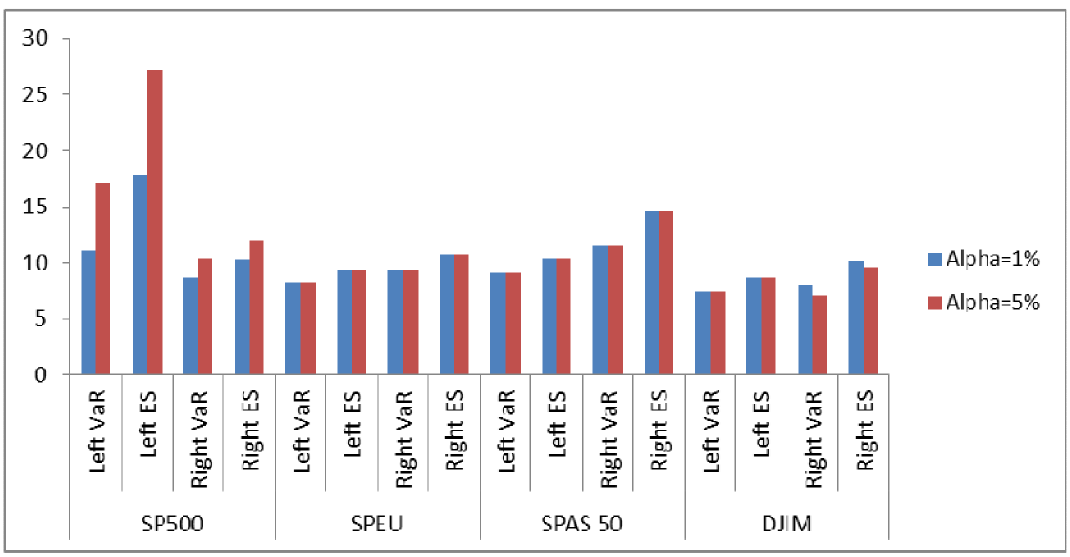

Figure 5: POT comparative risk measures.

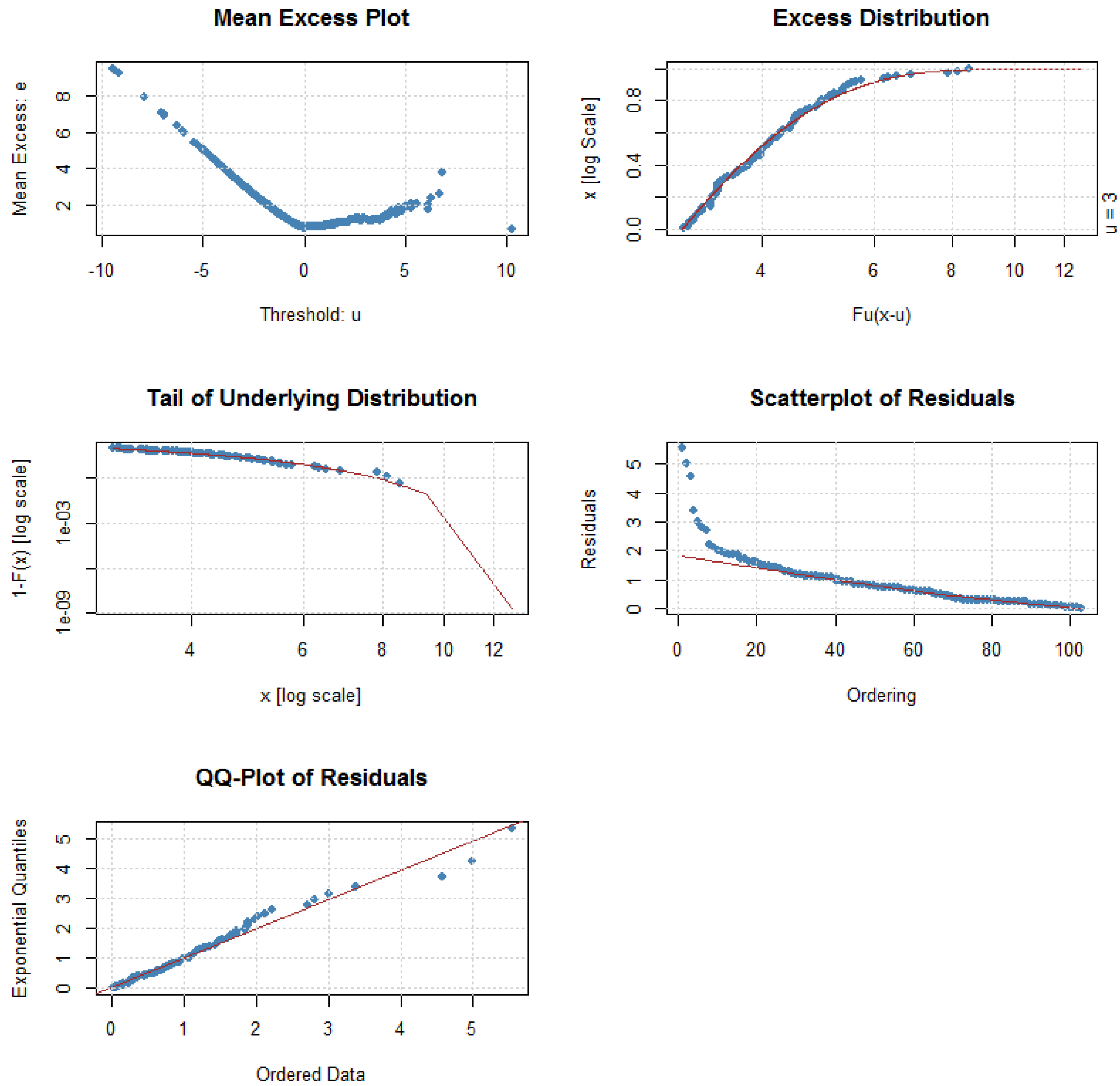

Figure 6: Checking the GPD for the SPAS50.

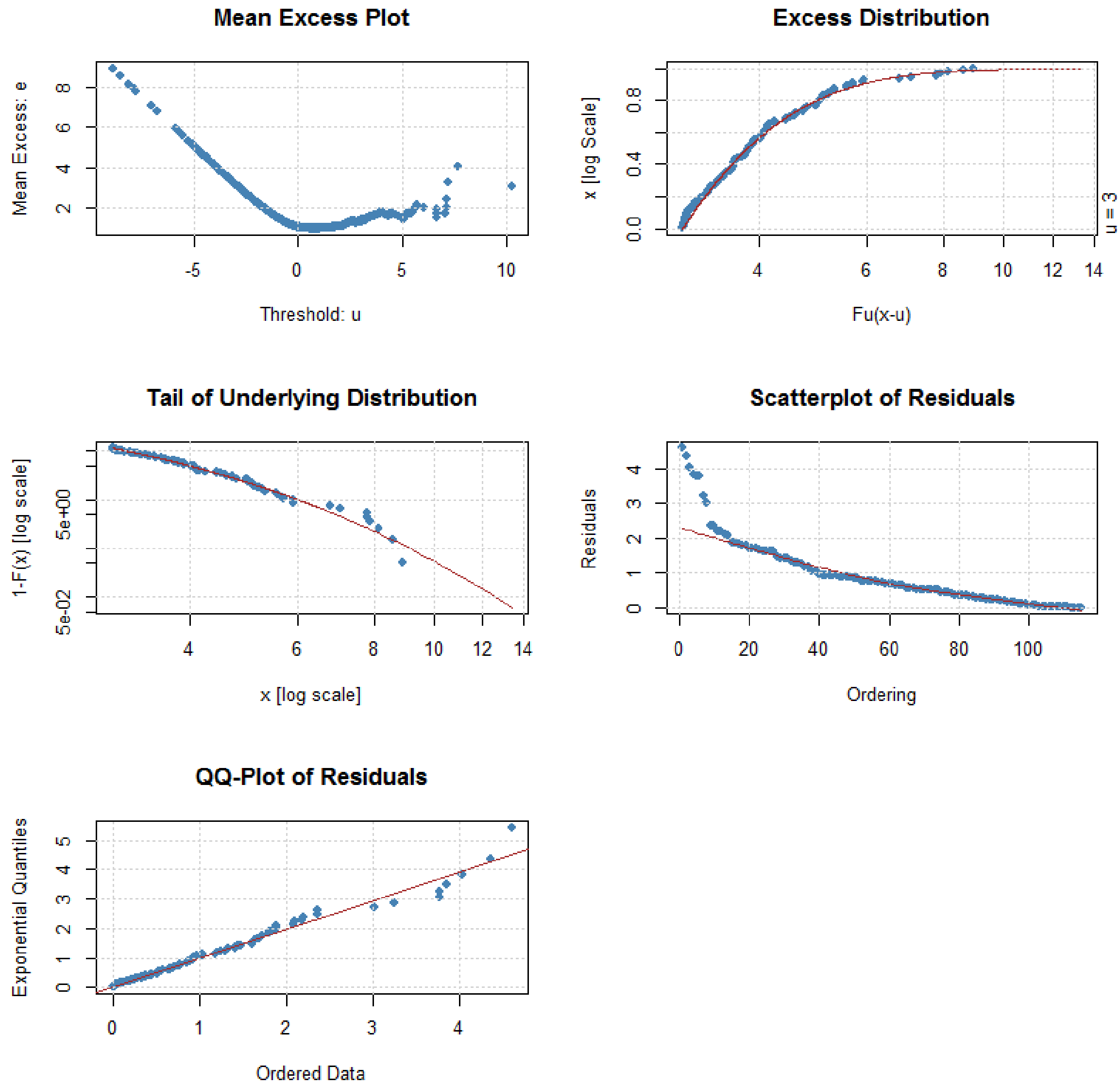

Figure 7: Checking the GPD for the DJIM.

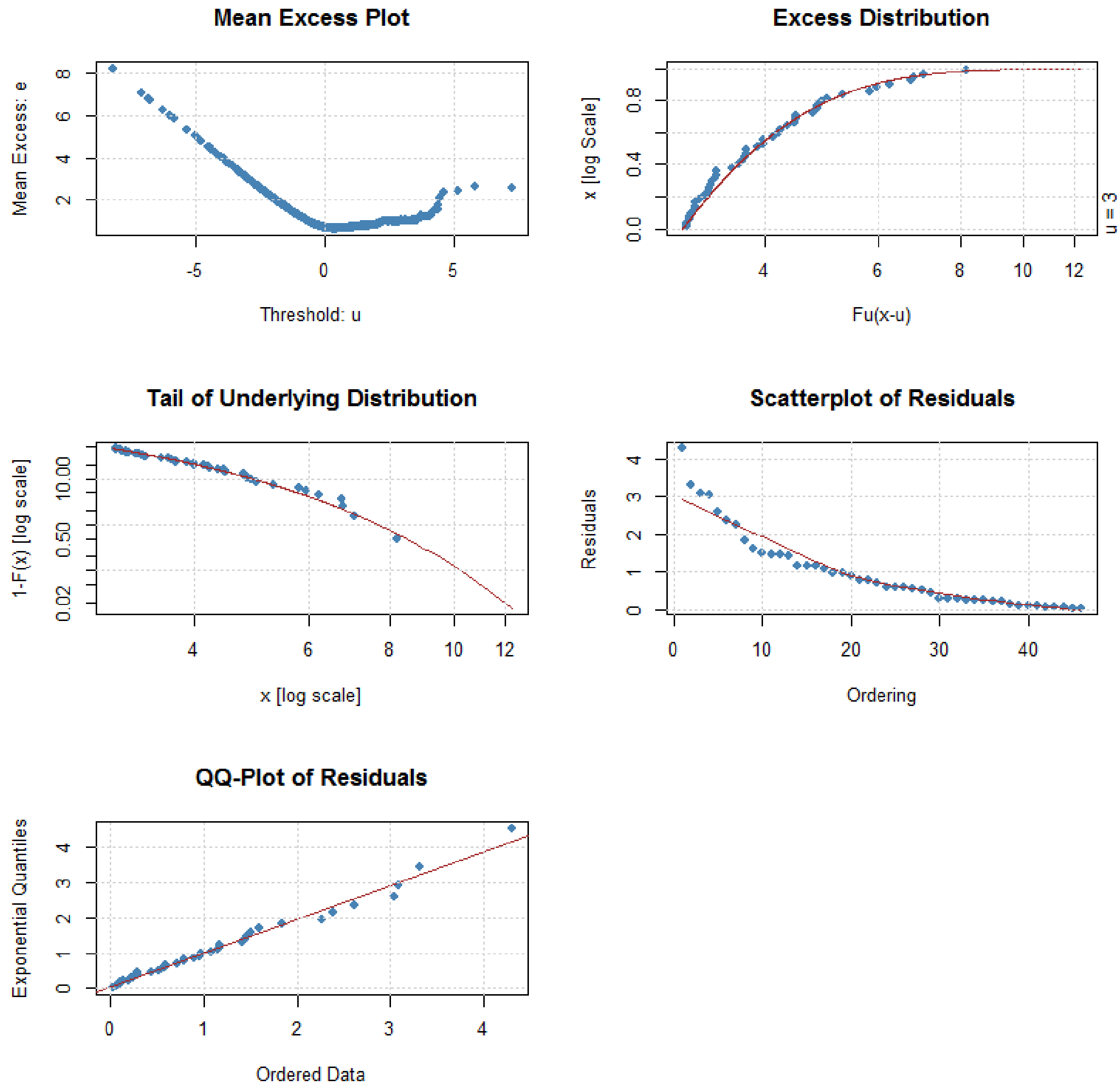

Figure 8: checking the GPD for the SPEU.

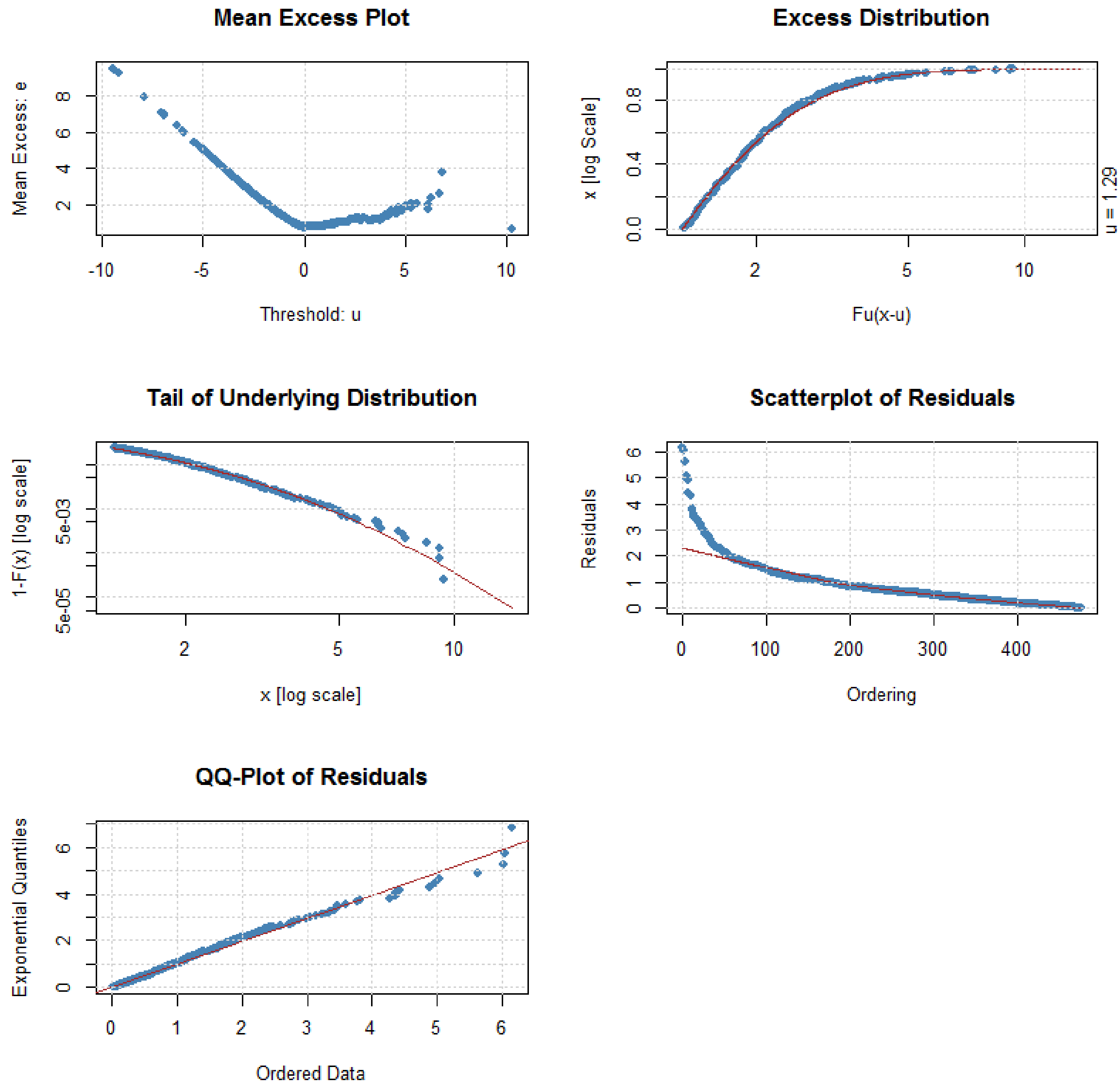

Figure 9: checking the GPD for the SP500.